Infinite Regress – Virtue or Vice?

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Infinite regress arguments are interesting for several reasons. They feature prominently in the philosophical literature. They are wielded, not only with great force, but also against a variety of positions and in an impressive number of different contexts – philosophy features few tools of equal range and power. Strong conclusions, such as those drawn from the premises of many infinite regress arguments, require strong justification. In this paper, I identify and investigate the reasons we should or must appeal to when arguing from the existence of an infinite regress to the rejection of the position from which the regress is derived.

An infinite regress argument necessarily includes, as one of its premises, an infinite regress. To be able to generate an infinite regress it seems as if at least the following ingredients are required (Gratton, 1997):

> *A regress formula*: Any statement (or combination of statements) that entails, or is intended by its author to entail, an infinite regress. The regress formula should be a recursive universally quantified proposition.
>
> *A triggering statement*: Any statement that serves as a trigger for the infinite regress.

To illustrate, consider the infinite regress of causes. In at least one version, the regress formula is, "everything has a cause" and the triggering statement, e.g. "*a* exists". From these ingredients, an infinity of causes can be derived. An infinite regress in itself neither

proves nor disproves anything; an infinite regress *argument* does.[1] To get an argument, we add to what is our main-premise (the infinite regress), a further set of independently obtained premises that, together with the generated infinity, entail a certain result. In the case of the infinite regress of causes, one well-known such premise claims that "actual infinity is impossible" (I will have more to say about the presumed impossibility of actual infinities below). This premise plus the infinite regress of causes, on at least one interpretation, entails the conclusion that *a* (or, anything else for that matter) does *not* exist. If we believe that this conclusion is unacceptable, the infinite regress argument should also include a third, and subordinate, argument showing *why* the conclusion obtained by means of, on the one hand, the generated infinite regress and, on the other hand, the (independently obtained) premise(s), is unacceptable. Adding such an argument is the same as saying "this regress is vicious" and that, consequently "the regress formula should be rejected".[2]

Subordinate arguments of this kind are often lacking in the actual application of the infinite regress argument. Instead, its use features a strange combination of self-sufficient determination ("this is the last word!") and lack of explicit justification.[3] That an infinite regress is vicious, or that this viciousness is sufficient reason for immediate rejection of the regress generating position is taken for granted or said to be "plain" or

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[1] Schlesinger (1983: 219): "There is nothing wrong with an infinite chain of something as such. No problem arises unless it is shown that to admit such a chain leads to some specifiable trouble."

[2] Other conclusion might be drawn, of course. It might be argued that it is the triggering statement that has to go (but this is rarely the case, the trigger is normally uncontroversial). Alternatively, it might be argued that the fault lies rather in some of the independently obtained premises other than our main premise. In an infinite regress argument launched *against* a position this is never the case. In arguments to the defence of some particular position charged with infinite regress this strategy is more common.

[3] Reflection is found in the lively *meta*-discussion of the argument, however. It is just that these reflections do not seem to have found their way into philosophical practice. Philosophers dealing with the argument from a meta-perspective include e.g.: Clark (1988); Day (1987); Gratton (1997); Johnstone (1996); Nathan,(2001); Nolan (2001); Passmore (1961); Priest (2002); Sanford (1984); Schlesinger (1983), and; Waismann (1968).

"obvious". A case in point is the following quote from Russell, who, after his production of a resemblance regress concludes that (1956:112, my italics):[4]

> [W]e explain the likeness of two terms as consisting in the likeness which their likeness bears to the likeness of two other terms, and such a regress is *plainly* vicious.

Such a blunt dismissal might be justified if a regress is, as Russell here seems to believe it is, *obviously* and *self-evidently* vicious. However, the fact that the viciousness of (almost) every infinite regress (including, as we shall see, *this* regress) can be debated, demonstrates that regresses are (almost) never self-evidently vicious.

An infinite regress is either vicious *or* virtuous.[5] Virtuous regresses are not virtuous in the sense that their existence is cause for celebration. Instead, they are virtuous either because you *can* tolerate them. One very good reason for assuming the existence of virtuous infinite regresses is the existence of what seems to be perfectly good examples of such. The truth regress, for instance, is considered quite innocent by most people and, if you are not one of those people, the arithmetic regress presents an even less controversial case.[6]

The Resemblance Regress

With terminological matters and one important assumption in place, let me now properly introduce my main-example – *the infinite regress of resemblances.* This regress appears in at least two contextually different infinite regress arguments: Russell's argument,

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[4] Here he complains that Resemblance Nominalism leads to an infinite regress of resemblances. I will return to the example discussed by Russell shortly.

[5] Some philosophers do not want to talk about virtuous *regresses.* To them, a regress is always vicious. Johnstone (1996:93) (using "benign" for my "virtuous"): "Since an infinite regress by definition is a *reductio* argument presupposing the denial of some form of axiom of infinity, there are no "benign" infinite regresses, asserting as they would have to *per impossibile*, the truth of such an axiom." Johnstone opts for the notion of a benign *series* for what will here be called a virtuous regress.

[6] The truth regress follows if it is accepted that if $<p>$ is some proposition then, if $<p>$ is true then $<p$ is true$>$ is true. The arithmetic regress, in turn, is a fact once it is accepted that every counting number has exactly one successor (distinct from itself and itself a counting number).

launched against the Resemblance Nominalist (RRR) and the infinite regress of resemblance-tropes discussed in contemporary trope-literature (TRR). The former regress is vicious and the latter is virtuous (or, so I will argue). To investigate the reasons we should or must have in order to, with justification, reject a theory because it generates an infinite regress means investigating the distinction between vicious and virtuous. A situation where what appears to be *the same* regress is vicious in one context and virtuous in another will therefore serve as an important illustrative purpose.

Russellian Resemblance Regress:

In "On the Relations of Universals and Particulars" (1911 (1956) the same argument can also be found in his 1912), Russell examines and dismisses what he calls "the theory which admits only particulars". The theory he has in mind seems to be the same as the one advocated by e.g. George Berkeley and David Hume in their polemic against abstract ideas. The theory is described by Russell as follows (ibid: 111):

> The general term 'white', in this view, is defined for a given person at a given moment by a particular patch of white which he sees or imagines; another patch is called white if it has exact likeness in colour to the standard patch. In order to avoid making the colour a universal, we have to suppose that 'exact likeness' is a simple relation, not analyzable into a community of predicates; moreover, it is not the general relation of likeness that we require, but a more special relation, that of colour-likeness, since two patches might be exactly alike in shape or size but different in colour.

To avoid making the relation of colour-likeness universal, the same analysis as was previously applied to the property shared by distinct objects must now be applied to it: "we may take a standard particular case of colour-likeness, and say that anything else is to be called a colour-likeness if it is exactly like our standard case." (ibid) To say this, however, leads to the infinite regress which, as we saw before, Russell believes to be "plainly vicious". The Russellian resemblance regress is discussed and its conclusions are accepted by Armstrong (1978). He is more precise in his explanation of why this regress is vicious; he writes (1978: 55):

> The original type, the property of *whiteness* is got rid of, but at the cost of installing another type, the relation of resemblance. Any attempt to get rid of the latter installs another type, a higher-order relation of resemblance (or perhaps further instances of the original relation). At each stage, therefore, the right-hand side of the analysis involves an unanalyzed *type*.

Perhaps Armstrong's interpretation of Russell's argument is not one with which Russell would fully agree. Here, this does not matter, the infinite regress, both as expressed by Russell and as interpreted by Armstrong, is vicious.

Trope-Theoretical Resemblance Regress:

According to a majority of (contemporary) trope theorists, tropes are not only one of the world's categories – it is its *only* category.[7] Assuming trope monism has many consequences, and one is that the trope theorist, unlike the universal realist, seems unable to account for the fact that two concrete particulars can exemplify *the same* property with recourse only to the property involved. The realist answers the question of what makes it true that a particular exemplifies a property and the question of what makes it true that two distinct particulars exemplify the same property by for both cases pointing to the instantiation of a universal. Universals both characterise individual particulars and explain their similarity/sameness to other particulars – all thanks to their universal nature. The same is not true of tropes. If two distinct concrete particulars, *a* and *b*, are both red then what makes it true that "*a* is red" is, according to trope theory, that *a* contains a particular trope t_1, and what makes it true that "*b* is red" is that *b* contains a particular trope t_2, where $t_1 \neq t_2$. From the non-identity of t_1 and t_2 it seems to follow that their existence is insufficient to make true the proposition that "the properties of *a* and *b* are *the same*".[8] Sameness cannot, as for the universal realist, equal numerical identity. An alternative explanation of property sharing is needed.

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[7] There are exceptions. One who believes that there are tropes *and* universals is C. B. Martin (1980; 2003).

[8] There *are* those that claim that it *is* sufficient (myself included 2002). However (and not counting Stout (1921)), those who think so, do it because they believe that the existence of the tropes is enough to account for the existence of *resemblance* (and that, consequently, resemblance is a "free lunch"). Holding exact resemblance to be a free lunch is (possibly) a way to avoid ending up in infinite regress in the first place.

According to the most common alternative explanation, what makes it true that two distinct concrete particulars share a property (e.g. are both red), is that each particular contains a red-trope and that the red-tropes *exactly resemble* one another. The exact resemblance of the red-tropes must, however, be given an ontological account. Given trope monism, it seems as if the only available ontological characterisation is one according to which exact resemblance is *yet another trope*. Unfortunately, this admission forces the theory into an infinite regress of the same type as that launched by Russell against classical Nominalism. To illustrate, imagine a situation where three objects share the property of being red. According to trope theory, this is so because each of the objects contains a red-trope and because the red-tropes belong to the same similarity class – because they *exactly resemble* one another. If there are only tropes, exact resemblance is a trope. Consequently, for each class of exactly resembling tropes, there will be one resemblance, a resemblance-*trope*, uniting its members. Moreover, all resemblance-tropes will be *the same*. As it has already been decided that to be the same is to be related by exact resemblance, the sameness of the exact resemblance-tropes will force into existence a second "level" of exact resemblance tropes. Resemblance-tropes that hold between resemblances are (also) the same – they are resemblances – i.e. (as above) there are resemblance-tropes holding between the resemblance-tropes, etcetera *ad infinitum*.

Many Distinctions – Or One?

To be able to argue that the Russellian regress is vicious whereas its trope-theoretical counterpart is not, we must agree on what *makes* a regress vicious (or virtuous). Most of what has been written about the distinction between vicious and virtuous has one important idea in common. It is the idea that, on closer inspection, the distinction will turn out to be not one, but several. This supposedly explains cases of mistaken use of the infinite

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This alternative is irrelevant to the present text, however, since here I aim to argue that even if a regress of exactly the kind envisaged in the argument *is* generated, trope-theory is not necessarily in serious trouble, since this regress is arguably not vicious.

regress argument (cases where a regress is said to be vicious, although it is not) – mistakes of this kind are due to insufficient sensitivity to the finer distinctions between different understandings of "vicious" and to the context of use. I will now argue the opposite: there is just *one* (interesting) albeit sophisticated and context-sensitive understanding of the distinction. To be able to explain how the (RRR) can be vicious when the (TRR) is not, it is on the details of this particular understanding of viciousness we must focus.

In the literature several different understandings of viciousness are suggested and discussed. These are all, and in line with the common belief, treated as substantially different yet equally valid. How these understandings are best characterised and under which circumstances their application is most suitable is, of course, debated. These suggestions can, I believe, be interestingly subdivided into *substantive* (here including the impossible and the idle understanding), *foundational* (the contradiction and the non-factual understanding), and *procedural* (the dependence understanding) accounts of viciousness: [9]

Substantive	Foundational	Procedural
Impossible	Contradiction	Dependence
Idle	Non-factual	

Substantive understandings situate viciousness in some property had by the generated infinity (either by the entities individually or as a group). Foundational understandings "blame", instead, some feature had by the theory from which the regress originates. Procedural understandings, finally, focus on the *manner* in which the regress is generated – its regress generating *mechanism*.

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[9] The table features most of the understandings discussed in the literature (although sometimes under a different name) and is meant to be uncontroversial. It may of course be that some interesting understanding that is different from the one's here offered is missing. Hopefully, any such omission will not matter here.

In what follows, I will argue, first, that viciousness *can* be understood in foundational terms, although not in an *interesting* sense. The reason why foundational understandings of viciousness are not interesting is that vicious regresses of this type always function as symptoms of an underlying disease, a disease that is unrelated to the infinite regress itself. Next, I will argue that viciousness *cannot* be understood in substantive terms. Any attempt to so understand it collapses the distinction and so contradicts the assumption that there *is* a distinction between vicious and virtuous regress. Finally, I single out the procedural so-called *dependence-understanding* as the only interesting one, and try to show how it can explain to us why the (RRR) is vicious, whereas the (TRR) is not.

Why a regress can be vicious because "Contradictory" or "Non-Factual" – and why this is uninteresting

Consider, first, the view that a regress is vicious if it is the consequence of some contradiction hidden in the theory from which it is generated. A possible example of an infinite regress of this kind is Plato's infinite regress of Forms (at least on one interpretation of it).[10] That an infinite regress derived from an inconsistent position must be considered vicious is trivial – for clearly, an infinite regress generated from a contradictory position is a *very* sure sign that the position in question should be rejected. What makes it such a sure sign, however, does not seem to be the generated regress – the regress is rather a symptom of an underlying disease. It is because the theory is inconsistent, and not be-

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[10] (Plato, *Parmenides*) Nolan (who actually does not himself endorse this particular reconstruction of the Platonic argument and uses it for illustrative purposes only) discusses this particular interpretation in depth (2001: 525f). It is proposed (but later given up) by Vlastos (1954: 324f). The inconsistency in question is, according to Vlastos, this: (i) there is a single form in virtue of which we apprehend things as F; (ii) this single form is itself F, and; (iii) this F-ness cannot be the Form in virtue of which we apprehend things as F. This means that F-ness is "both the single Form in virtue of which we apprehend things as being F, and is distinct from a Form in virtue of which it is apprehended as F. It is not identical, therefore, to itself." (Nolan: 528)

cause an infinite regress can be generated from it, that the position should be rejected. P. T. Geach puts the intuition this way:[11],[12]

> …often when philosophers *think* the trouble is a vicious regress, the *real* trouble arises already at the first step: if it is rightly diagnosed here, we can forget about the regress.

If the viciousness of a regress is due to the inconsistency of the position from which it is derived it is always *redundant* to point to the regress as grounds for rejection, even if regress obviously has some role to play in what we may call the "context of discovery". It is in this sense that the contradiction-understanding does not have anything interesting to tell us about the infinite regress – even if it does have something to tell us about why we find some theories from which an infinite regress can be generated, suitable for rejection.

That the same is true of the non-factual understanding is less apparent but, I think, nevertheless true. The non-factual understanding tells us that an infinite regress is vicious because the domain to which the generated entities belong is *in fact* finite, and known to be so. Possible examples are regresses that force one to postulate infinitely many actual beliefs, or regresses that demand an infinity of actual intentions. The non-factual understanding is, like the contradiction understanding, an *uninteresting* understanding, because, either, what is wrong with infinite regresses of this type is that the regress formula (e.g. "every action requires an intention") contradicts some other part of our theory (such as: "there are not infinitely many actual intentions"), in which case, the non-factual understanding is straightforwardly the contradiction understanding. Or, what is

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[11] P. T. Geach (1979: 100f.), quoted in Day (1987: 158). Similar views are represented by, among others, G. E. Moore (1962: 109f); Passmore (1961: ch. 2); and Sanford (1984: 94f).

[12] That regresses of this kind are not vicious (that they are, rather, symptoms of some underlying evil) is tentatively contested by Nolan (2001: 525f). According to Nolan, there are at least two reasons for not discounting these regresses: (i) regresses of this kind are normally treated as *vicious* regresses, and it is unnecessary to challenge normal usage on this point; (ii) (and more substantially) there is some reason to believe that the symptom/disease metaphor separates the regress a little bit too much from the contradiction – instead of thinking of one (the regress) as a mere consequence of the other (the contradiction) one should perhaps think of both as different ways of bringing out unacceptable features in a theory, neither of which takes precedence over the other.

wrong does have something to do with the infinite regress itself – but, what? It cannot be that the infinity in question contradicts things that we can empirically ascertain (how do you count the number of intentions around you?). The purported *fact* of finity can only be *theoretically* underpinned. From a theoretical perspective we might refer to the absurdities involved in the performance of infinitely many tasks (such as the production of infinitely many intentions) by finite beings in a finite amount of time (so-called "supertasking" – something I will return to below). However, if supertasking is the problem then (once again) the infinite regress is not (it is a symptom rather than a disease).

Foundational understanding of viciousness are not, as we have seen, *interesting* understandings of viciousness. Substantive understandings of viciousness, on the other hand, *are* interesting. By blaming a feature instantiated by the regress itself, they make the regress the disease rather than one of its symptoms. Unfortunately, they are not *possible* ways of understanding the distinction. They fail because they end up collapsing the distinction they are supposed to underpin.

Why a regress cannot be vicious because impossible

Consider, first, the substantive understanding of vicious, according to which, to say of an infinite regress that it is vicious, is to say of the infinity it generates that it is *impossible*. The infinity is impossible, furthermore, because it is *actual*. Although, as I will now try to show, infinity (actual or not) cannot be what distinguishes vicious from virtuous regress, there are fully comprehensible reasons for why one should think it would be. First, there is intuition (a powerful indicator in these circumstances). Most would agree, I think, that the very fact that a regress is *infinite* should constitute some grounds for caution. As put by Johnstone (1996: 97-98):

> Formulations of both nonvicious and vicious regresses may make use of the phrase "*ad infinitum*" (ειζ απειρον). In both cases, the hearer is supposed to regard this phrase as a danger signal – a warning of the same magnitude of seriousness as the phrase "…is a contradiction".

Exactly why "*ad infinitum*" signals danger, one can only speculate. Perhaps it is because the finite "is a basic fact of human existence", as Priest puts it. Priest continues (Priest 2002: 3):

> Whether one treats this as a source of sorrow or relief, it is without doubt that there are limits to whatever people want to do /…/ What these limits are, we can sometimes only speculate; but that they are there, we know.

That infinity is a red blanket is consequently no big mystery, and this may at least partly be what explains the tremendous influence that the impossibility understanding has had on our thinking about infinite regress and on infinity in general. But how do we get from regarding the presence of infinity as a danger-signal to regarding it as impossible?

One historically extremely influential reason for making this move was first formulated by Aristotle (*Physics*).[13] He argued that the existence of an actual infinity is impossible because it entails the existence of something with proper parts the same size as the whole to which they belong, and nothing can have parts the same size as the whole to which they belong because this would contradict the, according to him, *axiomatic* Euclidean principle that *the whole must be greater than its proper parts*.[14] Some infinities *are* possible, Aristotle admits. Possible infinity he calls *potential* (in medieval terms: syncategorematic). It is infinity such that (*Physics*, III.6 206ª27-206ᵇ2):

> In general, the infinite is in virtue of one thing's constantly being taken after another – each thing taken is finite, but it is always one followed by another; but in magnitudes what was taken persists, in the case of time and the race of men things taken cease to be, yet so that [the series] does not give out.

An actual infinity (a categorematic infinity) is a completed infinity, all of which members exist simultaneously. A potential infinity, on the other hand, only entails the existence of finitely many entities (at a time). The notion of an actual infinity is ontologically

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[13] Aristotle presented several reasons for his contention that actual infinities are impossible. Here I will only discuss one of these. The remaining reasons cited by Armstrong have not been treated kindly by time and the advances in science.

[14] Aristotle, *Physics* 1.4.188ª 2-4 ("…the unlimited number of corporeal particles would each contain an unlimited supply of flesh, blood and brain, <not> indeed separated from one another, but none the less real and unlimited; but that is nonsense"). For the Euclidean principle see: *Euclid* (Book 1, Common Notion 5).

"charged" – an actual infinity, if such there is, belongs to the furniture of the universe. Potential infinity, on the other hand, is infinity more in the sense of "capacity". The notion of a potential infinity is consequently an ontologically less expensive notion such that, if all infinity is potential (*Physics*, III.6 206^b 33-34):

> [i]t turns out that the infinite is the opposite of what people say it is: it is not that of which no part is outside, but that of which some part is always outside.

Could the distinction between a vicious and a virtuous regress be drawn along the same lines as that between an actual and a potential infinity? No, and there are at least two reasons why not. First, because it would seem that making actual infinity vicious-making collapses the distinction in at least some philosophical contexts. In a philosophical context where the sole concern is existence (an *ontological* context), any infinity generated in an attempt to account for *what there is* must be an actual infinity, as any such regress must be one generating *the existence* of infinitely many entities.[15] If actual infinities are impossible then at least ontological regresses are *always* vicious. Second, it is today unanimously agreed that the distinction cannot be drawn along the same lines as that of actual and potential infinity for the reason urged by Aristotle. Actual infinity is not impossible because it implies a whole which is equinumerous with one of its proper parts, because an infinity *can* be equinumerous with one of its proper parts. What broke the Aristotelian spell was an entirely new understanding of the nature of infinity that can be traced back (at least) to the publications of Georg Cantor toward the end of the 19th century (Cantor, 1932).[16] Cantor thought that the reason philosophers and mathematicians had had such problems with the infinite sets and their properties, was that they had all tried to understand infinity along the same lines as they already understood finity – and had failed mis-

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[15] Perhaps the same is true of infinite regresses generated in *all* types of philosophical contexts; I will not discuss that here.

[16] History was not completely devoid of defenders of actual infinity before Cantor, however. Leibniz was well known for his espousal of actual infinity. In a letter to Foucher, Leibniz wrote already in 1693 that: "I am so in favour of the actual infinite that instead of admitting that Nature abhors it, as is commonly said, I hold that Nature makes frequent use of it everywhere, in order to show more effectively the perfections of its Author. Thus I believe that there is no part of matter which is not, I do not say divisible, but actually divided; and consequently the least particle ought to be considered as a world full of an infinity of different creatures."

erably. Cantor instead introduced the surprisingly obvious idea that infinite and finite sets are quite different kinds of sets with radically different properties and pointed out that what it is to *be* an infinite set (as opposed to a finite one) is (among other things) to be such that the whole set can be equinumerous with one of its proper sub-sets. That which had made his predecessors deem actual infinity impossible, Cantor singled out as its distinguishing mark.

The question is now this: Does it follow from the fact that we can no longer say that actual infinities are impossible for Aristotelian reasons, that we cannot say that actual infinities are impossible (and that, consequently, the distinction between vicious and virtuous can be drawn along the same lines as that between actual and potential infinity)? No. Before finally rejecting the impossibility understanding as unfeasible we must investigate what other reasons one may have for deeming actual infinities impossible (or, at least problematic enough to make us deem regresses that generate infinities of this type vicious). That such reasons exist is clear from the lively discussion of the matter *after* Cantor had published his results. It was not so much the consistency of the notion, as the *role* that infinity was supposed to play, that was cause for criticism. One such critic, David Hilbert, argued that although mathematics, in a certain sense, can make good use of the notion of infinity, there is no actual infinity *in reality* (neither infinitely small nor infinitely large). Infinity, he claimed, still had a role to play, but only as an ideal addition to a finitist mathematics.

But, why should we not think that actual infinity could be real as well as ideal? There are several possible reasons. One, already mentioned, is our unfamiliarity with the infinite as opposed to the finite. This is not a *good* reason for banning actual infinity from reality, however, since it does not seem reasonable to let how well acquainted we are with something decide whether or not it exists. Another reason could be that we think that there is something about the real – reality – that makes it and unsuitable host for

actual infinity. To think otherwise, some would say, is absurd. An often cited example meant to illustrate this is that of the so-called paradox of the Grand Hotel:[17]

> An infinitely large hotel (a truly *Grand* hotel) with infinitely many guests (a "full" hotel, by finitist standards) can always fit one more guest in, by moving each of the guests already occupying a room to the room next to it (thereby leaving room one free for the newcomer). In fact, it can fit infinitely many new guests in (by, this time, moving each guest to a room with a room-number twice as large as the one they were occupying, thereby leaving all the odd-numbered rooms free for the infinitely many newcomers). And, if infinitely many guests move out – it will still be full!

William Lane Craig (1991: 85-96) thinks that the paradox of the Grand Hotel is *proof* that we should *not* think that real actual infinities are possible, just because ideal ones are.[18] He asks (ibid: 86):

> Can anyone sincerely believe that such a hotel could exist in reality? These sorts of absurdities illustrate the impossibility of the existence of an actually infinite number of things.

To ask questions is not to make an argument, however. *What* about the example strikes Craig as so absurd? Craig himself points to two things: the requisite addition of guests and the fact that the odd-numbered rooms should be as many as the total amount of rooms in the hotel. Neither absurdity will do for anyone wishing to argue that actual infinity in reality is impossible, however. The latter "absurdity" is quickly dismissed because, as we have seen, when it comes to infinity, our intuitions seem (wrongly) modelled on the finite. The first absurdity is more interesting. There *is* something troubling about the performance of the requisite room-changing task, but what? The absurdity in question can only work as an argument against the real possibility of actual infinity, if it is caused by the *existence* of an actual infinity. Is it? Moving guests in the way required involves a "supertask"; a task with infinitely many steps (the adequate changing of rooms) performed in a finite amount of time. Finite beings (such as the infinitely many

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[17] This is not really a paradox in the logical sense of the word – it is rather an example exploiting the extreme unintuitiveness of the idea of an actual infinity as applied to concrete reality.

[18] And he then goes on to apply this conclusion to the cosmological argument for the existence of God (God must exist, for if he does not an actual infinity of causes can be generated into the past, actual infinities are impossible, so there must be a first cause and this is God).

guests occupying rooms at the hotel) could most probably *not* perform the infinite task of changing rooms in a *finite amount of time*. Even if we agree that supertasking involves us in absurdity (in at least some sense of that word), this does not mean that the actual existence of infinitely many physical (real) entities is impossible.

Craig's reasons will not do, but there are other suggestions. Maybe we should not look for an explanation of why reality cannot harbour infinity by studying the properties of infinity, but rather by studying the properties of the entities of which the real (as opposed to ideal) infinities are composed. Ideal infinities involve infinities of *abstract* entities (such as numbers). Real infinities, on the other hand, seem to involve sometimes abstract, sometimes concrete entities. An infinity of resemblances, or an infinity of ideas, is an infinity of *abstract* entities. An infinity of causes, however, might be one including only concrete entities. Maybe actual infinity is vicious-making only if it is an actual infinity of *concrete entities*. But what makes a real infinity of concrete, as opposed to abstract, entities so problematic? Some would perhaps say that real actual infinities of concrete entities are (physically) impossible because of the worldly "overpopulation" they result in. There is no room for all these entities. As the size of the universe seems up for grabs, we may therefore reasonably ask: If the universe is finite in size, could it make room for an infinity of concrete entities? Russell, discussing these matters in *The Principles of Mathematics,* says 'yes'. As long as we believe that there are bounded stretches of space (or time) it in fact follows that there are actual infinities in the world. He points out that those who deny the existence of actual infinity still admit that what they call finite space may very well be a "given whole", but (1903:144):

> …such a space is only finite in a psychological sense – it is not finite in the sense that it is an aggregate of a finite number of terms, nor yet a unity of a finite number of constituents. Thus to admit that such a space can be a whole is to admit that there are wholes which are not finite. With respect to time, the same argument holds.

This seems reasonable enough, and so, not even the idea that *some* real actual infinities are impossible, can be maintained. There *is* another reason for holding that it is the infinity itself that is the problem, but which does not necessarily entail the *impossibility* of

whichever infinity we are discussing. This reason has been much debated (but not endorsed in its most radical form) by Daniel Nolan. It says that, an infinite regress is vicious because it entails infinity, and infinity is vicious-making because it violates the principle of ontological parsimony.

Why a regress cannot be vicious because ontologically extravagant

For someone wishing to argue that there is, after all, something about infinity itself that is problematic, all is not lost. There *is* another, often cited, reason for why infinity is, if not *impossible*, then at least seriously problematic, which is echoed in the so-called idle understanding of viciousness. On the idle understanding, an infinite regress is vicious because it entails infinity, and infinity is vicious-making because it violates the principle of ontological parsimony. To understand the distinction between vicious and virtuous in terms of ontological parsimony is to make use of "Ockham's razor". Once again, it is the *number* of entities that is in focus (notice that the razor employed cannot be of Lewis' kind – which only shaves off kinds of entities – it must also endorse economy in the number of entities of each kind). It is important to realize that the question of whether Ockham's razor can help us decide if a regress is vicious or not does *not* depend on whether the ideal of simplicity and parsimony it expresses is a virtue. Simplicity and parsimony *are* theoretical virtues. All else being equal we should always prefer the theory that to a higher degree respects these ideals. Even so, Ockham's razor *cannot* provide the criteria by which we distinguish a vicious from a virtuous regress. Why not? The simple (some would perhaps say simplistic) answer is that the number of entities involved in every case of infinite regress (whether vicious or virtuous) is always the same – *infinite* – and so number of entities cannot be what decides either way. It *can* be what explains our feeling of discomfort when it comes to *all* regresses – they will all sin against what I agree is a theoretical virtue. This is noticed by Nolan, who argues that (2001:537-8):

> … in recognizing that there is a genuine cost involved even in "benign" infinite regresses, my theory provides more of an explanation for why some people find even these benign regresses unpalatable – they are making an error, by my lights, but it is an error of taking a theoretical cost to be more expensive than they should, rather than the more curious mistake of seeing a cost where there is none at all.

Pointing out that the razor cannot distinguish between vicious and virtuous regress because both types of regress are infinite is not enough to finally rule out the idle understanding, however. The razor, its defenders argue, is more sophisticated than that. What it tells us is that we should not postulate entities *beyond necessity*. Could not the distinction between vicious and virtuous be one according to which a vicious regress postulates entities beyond necessity whereas a virtuous regress does not? This (or some version of this) seems to be Nolan's view when he says that (2001: 536-7):

> My suggestion is that the boundary might well be this: a regress is taken to be benign when the quantitative extravagance is a cost worth paying, and vicious when either the quantitative extravagance is not a cost worth paying, or if it has some more serious fault of which the regress is evidence (like lurking contradiction or failure as a reductive analysis).

To be able to use this more sophisticated razor, we must be able to decide when the entities generated in the regress are necessary and when they are not. It is often said that unnecessary entities are entities that are *idle* or *inert*. The razor is then interpreted as prescribing us not to postulate idle entities. The trouble is that, once again, this type of criteria seems unable to distinguish what we feel are vicious from virtuous regresses. What does it mean for an entity to be idle? A plausible answer is that an entity is idle (and hence unnecessary) if it exists, not for some particular purpose but only because a certain theory necessarily and automatically generates it. Now, obvious cases of (in this sense) *vicious* regress turns out to be the classical *virtuous* ones: e.g. the truth-regress and many of the mathematical regresses. Worse, what *would* be a virtuous regress on this account? Infinite regress in general is characterizes by its compulsive production to infinity for no other reason than the existence of a mechanism unrelated to the issues the theory in question is meant to address. Does this not make *all* infinite regresses vicious?

Nolan might object that this is not the criteria proposed by him. His criteria is formulated in terms of costs and benefits and says that a virtuous infinite regress is one where the cost of quantitative extravagance is acceptable and a vicious is one where it is not. I think he is right to think that the standard of parsimony (whether quantitative or qualitative) is not absolute. It will always be relative other standards and so its weight can never be decided until other factors have been taken into consideration. However, and for the same reason, quantitative parsimony *cannot* be what distinguishes vicious from virtuous regress even if it *can* be what distinguishes good from bad (or less good) theories.

The dependence understanding

We come now to the understanding of viciousness which I believe is the only one that manages to stay interesting, while not ruling out a substantial distinction between vicious and virtuous infinite regress in *any* philosophical context: *the dependence understanding.* The dependence understanding is not substantive – it does not blame the viciousness of the regress on any particular feature instantiated by the entities it generates. It is not foundational – it does not blame the viciousness of the regress on something having to do with its origin. It is, instead, procedural. It singles out the regress-generating *mechanism* as sometimes the source of viciousness. It is by focusing on a feature that is intimately tied to the infinite regress itself (albeit not instantiated by the entities generated by it) that this account manages to avoid the charge of redundancy.

But how can studying the regress generating mechanism be enough to identify the vicious making features we are searching for? Are not vicious and virtuous regresses generated by exactly the same kind of mechanism? True, I have so far proceeded under the assumption that vicious and virtuous infinite regresses are generated in exactly the same way. Now, however, I will explain why this was not exactly right. Important here will be the relationships that hold between the different "levels" of a regress – in particular, what I will call their "direction". Important is also the philosophical context in which

a regress is generated. The context will decide how the regress mechanism (and the relationships between its different levels) should be understood. This is why what may seem to be the same resemblance-regress, because it is generated in different philosophical contexts, will become vicious in one and virtuous in the other. The dependence understanding departs from the observation that every *statement* of an infinite regress seems to instantiate the same pattern:

1. Some question is posed/some problem is formulated.
2. Some answer is given/some solution is provided.
3. It is shown that the type of question/problem asked/formulated in the first step can now be asked/formulated once more – this time for the answer /solution given in step (2).
4. The new question/problem is answered/solved (in the same way as in (1)).
5. It is shown that the type of question/problem asked/formulated in the third (and first) step can now be asked/formulated once more. . . .
6. etc. *ad infinitum*

In fact, this pattern is the same for both vicious and virtuous regresses. If the pattern is interpreted as showing you that you are faced with an indelible problem the regress is deemed vicious, but if it is interpreted as showing you that every time you ask a question you can come up with an answer, the regress is said to be virtuous. In straightforward cases, we seem to know intuitively when it is appropriate to go with the first interpretation and when to go with the second. The dependence understanding departs from the assumption that the familiar pattern of infinite regress statements tells us something about the infinite regress itself.

Dependence and direction

As the pattern looks the same whether or not the regress is vicious it seems as if what decides whether we should interpret it as a pattern with eternally recurring questions or one with eternally recurring answers, must be something that can be "read between the lines". The dependence understanding treats this expression quite literally and looks for

an explanation in the relationships that hold between the different steps in the regress pattern.

The best and most neutral way to spell out these relationships (a way of spelling them out that will work whether or not we are talking of the relationships between the different steps of a regress *statement* or of the infinite regress itself) is in terms of *dependence* – and *direction of dependence*. An early formulation of what seems to be the same idea is found (as so many times before) in Russell's work. He argues (1903: 348) that "in the objectionable kind [of regress] two or more propositions join to constitute the *meaning* of some proposition; of these constituents, there is one at least whose meaning is similarly compounded; and so on *ad infinitum*." And, again, "an endless process is not to be objected to unless it arises *in the analysis of the actual meaning of a proposition*" (ibid: 51). Objectionable regresses are contrasted with the *un*objectionable kind. Russell provides an example: "[i]f A be a proposition whose meaning is perfectly definite, and A implies B, B implies C, and so on, we have an infinite regress of a quite unobjectionable kind" (ibid: 349). A regress is not vicious, that is, if there is no logical necessity to complete the infinite regress before the concept in question acquires a meaning. In this sense, virtuous regresses are *implicative* whereas vicious regresses are *analytical*.

Russell's distinction is put in terms referring to meaning, analysis and implication. To acquire a more general application, it should be put in more topic-neutral terms – terms that are applicable also in contexts where what is at stake is *not* the fixation of meaning. It is this role that the notion of dependence is supposed to play. Put in dependence terms, we can now see that, when an infinite regress is vicious the direction of the dependence between its different steps is the opposite from what it is when the regress is virtuous. A vicious regress exhibits the following pattern of dependence:
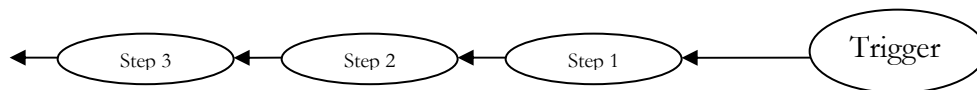
What I call the "trigger" is here supposed to be whatever the previously mentioned triggering statement refers to. For Russell, the trigger is some proposition whose meaning is supposedly fixed, but it could be almost anything. I could be some justified belief or some true proposition etc. The arrows indicate that the regress is vicious because the trigger, to exist (or, the triggering statement, to be true) requires, first, that step one exists (or, is true), which, in turn requires that step two exists (or, is true), etc. *ad infinitum.* The existence of the trigger will therefore depend on the existence of some "final" step of the regress – a step that will never exist given that the regress is infinite.[19] A virtuous infinite regress exhibits the opposite pattern of dependence (figure 5):
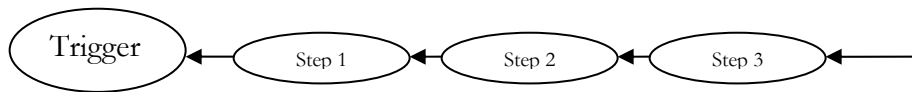
In a virtuous infinite regress, each step requires, for its existence, the existence of each previous step, and, ultimately, of what I call the trigger. The existence of the trigger in no way depends on the generation of infinite regress and neither does the existence of any of its infinitely many steps. The infinite regress is a mere side effect, automatically produced given the existence of the trigger. It is *because* of these differences in dependence patterns that we sometimes want to say that, at each step a question is posed, and sometimes that at each step an answer is provided. This pattern, furthermore, is completely general. No matter which context, no matter what trigger, what distinguishes a vicious from a virtuous infinite regress is its pattern of dependence.

Where, then, does context come in? Context comes in when what is required for the trigger to exist is to be determined. The context, the theoretical framework in which a particular philosophical theory is set, consequently plays a very important role when it comes to deciding whether or not a particular infinite regress is vicious or not. To know

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[19] Talk of "last" and "first" etc. is, of course, not entirely correct. If the infinity generated is actual, every step of the regress will obtain "at once". Yet, the problems pertaining to the direction of dependence will remain even in this case.

enough about the context to be able to determine what is required for the trigger to exist is far from easy, as will become apparent when we evaluate the difference between the (RRR) and the (TRR).

Evaluating the (RRR) and the (TRR)

On the face of it, the Russellian resemblance regress and its trope theoretical counterpart will look exactly the same. The trigger in both cases is the state of affairs that *a* exactly resembles *b*. One difference is, of course, the nature of *a* and *b*. On the view criticised by Russell, *a* and *b* are concrete objects, whereas the basic question for trope theory will concern the exact resemblance of tropes. The *relevant* difference is not this difference in nature, however.

To be able to determine the direction of dependence in each case we must look to context. According to the view criticised by Russell, for an object to have a property is for it to be exactly like the object serving as a standard for that property in question. According to trope theory, for an object to have a property is for it to contain a trope. Tropes belong to classes of exactly similar tropes, and because objects contain tropes, they too form classes based on their likeness in certain respects. However, contrary to with what is case for the view criticised by Russell, objects do not have properties *because* they belong to some particular similarity class. Instead, they belong to some particular similarity class, *because* they have some particular properties – the tropes. As a result, if you subscribe to the view criticised by Russell, then for the trigger – *a* is exactly similar to *b* – to exist (or, obtain), the existence of the similarity class, to which the exact similarity holding between *a* and *b* belongs, must also exist, and so on for each new level of exact similarity. If, on the other hand, you are a trope theorist of the kind here described, the same trigger, to exist, will in fact require no more than the existence of tropes *a*, *b* and their trope of resemblance. The fact that the view criticised by Russell requires, for

the existence of the trigger, that the next step of the regress should exist, and so on, *ad infinitum* makes this regress vicious. In the trope case, on the other hand, the infinite regress does not prevent the trigger from existing. It is rather the existence of the trigger that sets into motion the infinite generation of exact resemblance tropes. The trope theoretical resemblance regress is, therefore, virtuous.

The contextual differences that make for such different evaluations run even deeper. The view criticised by Russell is driven, I imagine, by the idea that the structure of language and that of the world is to a large extent the same. This is why a nominalist, with recourse only to particulars, must spend much energy trying to explain (or, explain away) the generality we find in language – especially when it comes to the terms we have for referring to properties and relations. The nominalist must get rid of all types and be left with only tokens. This is why the nominalist cannot settle, as I believe the trope theorist can, with the primitively natured objects invoked. Contemporary trope theorists, on the other hand, often champion the coming apart of truth and meaning. Yes, language contains generality for which trope theory will not always be able to provide the perfect substitute. This, however, does not matter since a metaphysical theory only has to include enough to account for truth by providing the minimal amount of truthmakers. To make true '*a* and *b* are both red' the existence of *a* and *b* is required. From their existence, their resemblance follows necessarily – and so will not make a separate contribution to truth. Another way of putting this is to say that the steps following the trigger will *supervene* on it. Some claim that, from this observation follows a much better solution to the regress problem than the one suggested here. If all the steps of the regress supervene on the trigger then, there is in fact no regress for us to worry about – neither vicious nor virtuous. No infinity is ever generated since, it is argued, what supervenes is an "ontological free lunch". Is this a better solution? No. The dependence understanding has the advantage of being able to dismiss the infinite regress as harmless without having to appeal to *ad hoc* principles such as one holding that what supervenes does not exist.

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Hopefully you are now convinced that the distinction between a vicious and a virtuous infinite regress is one, simple, straightforward and conclusive. Viciousness is determined by direction of dependence. I hope that you are also convinced that *deciding* whether or not a particular infinite regress is or is not vicious is complicated indeed. To be able to decide either way, very fundamental methodological matters must be considered. Matters that are not always easy to identify and which bring with them their own trials and tribulations.

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