Pufendorf Lectures 2025: **Vocabularies of Reason**

Handout for Lecture II

Logic and the Structure of Reasons

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I. The Structure of Reason Relations

What is the relation between *material* reason relations of implication and incompatibility and *formal logical* relations of consequence and inconsistency?
The former includes implications such as
Pittsburgh is to the West of New York |~ New York is to the East of Pittsburgh.
It is raining |~ The streets will be wet.
These are good because of the content of the nonlogical concepts involved.

Material reason relations facially do not have the same structure as formal logical ones.
Logical consequence and inconsistency are monotonic and transitive.
Tarski's view is that the essential structure of consequence relations is that they are *topological closure operators*. In our terms, he takes a vocabulary to be a lexicon L of sentences, and a two-place relation between sets of sentences pairing each subset $X \subseteq L$ with its consequence-set Con(X), satisfying these conditions:
Containment (CO): $X \subseteq \mathrm{Con}(X)$.
Monotonicity (MO): $X \subseteq Y \Rightarrow \mathrm{Con}(X) \subseteq \mathrm{Con}(Y)$.
Idempotence (CT): $\mathrm{Con}(X) = \mathrm{Con}(\mathrm{Con}(X))$.
(These are variants of the Kuratowski axioms for topological closure operators.)

But materially good implications can be nonmonotonic:
Tweety is a bird |~ Tweety flies.
Tweety is a bird and Tweety is a penguin # Tweety flies.
And we can turn failures of monotonicity into failures of transitivity:
Tweety is a penguin |~ Tweety is a bird, Tweety is a bird |~ Tweety flies
These two premises do *not* entail: Tweety is a penguin |~ Tweety flies.

Cautious Monotonicity (CM): $\dfrac{\Gamma \mid\!\sim A \quad \Gamma \mid\!\sim B}{\Gamma, B \mid\!\sim A.}$

Cumulative Transitivity (CT): $\dfrac{\Gamma, B \mid\!\sim A \quad \Gamma \mid\!\sim B}{\Gamma \mid\!\sim A.}$

If Γ|~A and G∈Γ, then G is part of the *explicit* content of Γ, and A is part of the *implicit* content of Γ--in the literal sense of being *implied by* Γ.
Explicitation is moving a sentence from the conclusion side of an implication to the premise side. This is the paradigmatically rational activity of explicitly acknowledging the consequences of one's commitments.
Since CM says that explicitation never *subtracts* consequences from a premise-set, and CT says that explicitation never *adds* consequences to it, together they entail that ***explicitation* is *inconsequential***. But explicitation is *not* always inconsequential. Example: database of observations plus theory as inference engine to extract consequences.
So some implication relations are *hyper*nonomonotonic: neither topologically closed nor explicitationally closed.
In these radically substructural settings, extracting consequences by explicitation is path dependent: it matters in what order one extracts consequences. There need be no such thing as *the* unique set of consequences of a premise-set.

II. Expressivism and Logic

The *reasons question* about logic asks: What is the relation between logic and reasons?
Logicism about reasons: Good reasons are always *logically* good reasons.
Structural logicism about reasons:
The structure of reason relations is the topologically closed structure of *logical* reason relations.
Logical expressivism: The expressive role characteristic of logical vocabulary is to make explicit reason relations of implication and incompatibility.
Expressive goal of *structural universality*: The reason relations codifiable by expressively ideal logical vocabulary include not only topologically open (nonmonotonic and nontransitive) ones, but also explicitationally open ones (hypernonmonotonic reason relations, in which even CM fails).
The expressively ideal logic is *universally LX*: it can be conservatively *elaborated from* (L) and is *explicative of* (X) any constellation of reason relations whatsoever, regardless of its structure.

Sequent calculi consist of metainferential rules that show how to extend the reason relations to correspond to an extension of vocabulary by introducing logical operators.
The operational rules do that, in the context of the structural rules.

Deduction-Detachment (DD):

$$\frac{\Gamma, A \mathrel{|\!\sim} B, \Delta}{\Gamma \mathrel{|\!\sim} A \rightarrow B, \Delta.}$$

Bidirectional Meta-Inference Line

Incoherence-Incompatibility (II):

$$\frac{\Gamma, A \mathrel{|\!\sim} \Delta}{\Gamma \mathrel{|\!\sim} \neg A, \Delta.}$$

Bidirectional Meta-Inference Line

There are many different formulations (for instance, in sequent calculus MVs) that are equivalent in that they all specify the consequence relation of classical logic. Under substructural conditions, they come apart, and specify different consequence relations.
One of them—which is just a formulation of classical logic, under strong structural conditions—degrades gracefully when we go substructural.
It remains universally LX, and expressively complete in a very strong sense.

*N*on*M*onotonic*,M*ulti-*S*uccedent logic, or NMMS for short, has three remarkable properties.

1. It is *expressively complete* in an unprecedentedly strong sense. We show how to associate each sequent or set of sequents whose premise-set and conclusion-set consist of logically complex sentences with a set of sequents in the base vocabulary, which relate only the logically atomic sentences that occur in them, such that those sequents from the logical supervocabulary hold in all and only the NMMS-elaborations of bases in which just those atomic sequents hold. In this clear sense, the logically complex sequents *say that* the corresponding logically atomic sequents hold. It is LX for its base vocabularies.
2. The second remarkable property of NMMS is that it is fully tolerant of open-structured or radically substructural base vocabularies. This feature has to do with the *elaboration* dimension of LX-ness rather than the *explication* dimension, as the first point did. NMMS conservatively extends logically atomic base vocabularies that are nonmonotonic and nontransitive, those in which Cautious Monotonicity fails, and even those for which Containment fails. It is *universally* LX.
3. The third remarkable property of NMMS is that it is essentially just classical logic. In the fully topologically closed setting defined by Gentzen's full set of structural rules, NMMS yields exactly the same logically valid sequents as Gentzen's sequent-calculus version of classical logic, LK. NMMS is supraclassical when applied to any base vocabularies that include all instances Containment, and yields exactly the classically valid implications and incompatibilities if it is applied to base vocabularies *all* of whose implications are instances of Containment

<u>Connective Rules of NMMS</u>:

L¬: $$\frac{\Gamma \mid\!\sim \Delta, A}{\Gamma, \neg A \mid\!\sim \Delta}$$

R¬: $$\frac{\Gamma, A \mid\!\sim \Delta}{\Gamma \mid\!\sim \Delta, \neg A}$$

L→: $$\frac{\Gamma \mid\!\sim \Delta, A \quad B, \Gamma \mid \Delta \quad B, \Gamma \mid\!\sim \Delta, A}{\Gamma, A \rightarrow B \mid\!\sim \Delta}$$

R→: $$\frac{\Gamma, A \mid\!\sim B, \Delta}{\Gamma \mid\!\sim A \rightarrow B, \Delta}$$

L&: $$\frac{\Gamma, A, B \mid\!\sim \Delta}{\Gamma, A \& B \mid\!\sim \Delta}$$

R&: $$\frac{\Gamma \mid\!\sim \Delta, A \quad \Gamma \mid\!\sim \Delta, B \quad \Gamma \mid\!\sim \Delta, A, B}{\Gamma \mid\!\sim \Delta, A \& B}$$

L∨: $$\frac{\Gamma, A \mid\!\sim \Delta \quad \Gamma, B \mid\!\sim \Delta \quad \Gamma, A, B \mid\!\sim \Delta}{\Gamma, A \vee B \mid\!\sim \Delta}$$

R∨: $$\frac{\Gamma \mid\!\sim \Delta, A, B}{\Gamma \mid\!\sim \Delta, A \vee B}$$

Material is from Chapters 2 and 3 of Ulf Hlobil and Robert Brandom
Reasons for Logic, Logic for Reasons: Pragmatics, Semantics, and Conceptual Roles [Routledge, 2024].