# Four Brides for Twelve Brothers: How to Dutch Book a Group of Individually Rational Players [1]

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*You can't be wise and in love at the same time*. (Bob Dylan)

**Abstract.** Wlodek Rabinowicz suggested in an e-mail conversation (2001) to me that one might be able to use a particular Hats Puzzle to make a Dutch Book against a *group* of individually rational persons. I present a fanciful story here that has the same structure as Rabinowicz's Dutch Book.

Once upon a time in distant Thracia there lived a tribe that had large families and a high neonatal female mortality rate. This led to the practice of a curious kind of polyandry. It was the custom that a single girl from one family would invariably marry three brothers from another family. The parents of the bride were required to pay a dowry of one coin of gold to the family of the bridegrooms. The courting ritual had a precise timing and a definite structure. On Friday morning, the three brothers would propose to the girl whom

[1] In April 2001, I sent the *New York Times* article "Why Mathematicians Now Care About Their Hat Color" to Wlodek Rabinowicz. He countered in an e-mail with a variant on this puzzle suggesting that it may permit one to make a Dutch Book against groups of individually rational people. I present a fanciful version here of Rabinowicz's Dutch Book. Here is how Rabinowicz's Dutch Book runs: Suppose that we distribute white and black hats to a group of three people. There is a 50-50 chance that each person receives a hat of one colour or the other. Clearly, the chance that, (A) not all hats are of the same colour, is ¾. Now suppose that all persons can see the colour of the hats of the other persons, but not the colour of their own hats. Then no matter what combination of hats was assigned, at least one person will see two hats of the same colour. For her the chance that not all hats are of the same colour strictly depends on the colour of her own hat and hence equals ½. A bookmaker could make a guaranteed profit by (i) offering to sell a single bet on (A) that pays $4 and costs $3 before anyone is informed of the hat colours of the other people, and (ii) offering to buy a single bet on (A) that pays $4 and costs $2 after the players come to see the hat colours of the other players. If the players are willing to take on fair bets, any of the players would

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they had set their hearts on. Subsequently, they would return to their respective rooms and they were forbidden to have any contact with each other. On Saturday morning, the girl would slip notes under their doors saying either yes, no or maybe and she could say yes (or no) to some and maybe to others. Girls always followed up when they had given definite answers, but there was no telling what they would do when they had said maybe. By Sunday, the girl had to have made up her mind. If she had decided that the wedding was on, then she would go to the temple of Apollo and the brothers would be summoned.

There was a poor single mother with four charming daughters who each received weekly suitors. They begged their mother to let them marry, but the mother did not have a coin to spare. She had a degree in Casino Management from Athens University, but this did her little good in old Thracia. She had started a casino, which soon went bankrupt. It is not that Thracians did not like to gamble, but they were an extremely rational people: they take up all and only bets that are either fair or more than fair. Now casinos are not in the business of offering fair or more than fair gambles. The mother was also unable to keep a secret, which made it impossible to exploit any superior knowledge of empirical matters that she might have in setting up gambles. She also knew very well how to exploit patterns of synchronic and diachronic irrationality by means of Dutch Books. But also this did not do her any good, since Thracians don’t display a trace of synchronic or diachronic irrationality.

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be willing to buy the first bet and at least one player would be willing to sell the second bet. Whether all hats are of the same colour or not, the bookmaker has a guaranteed profit of $1.

So what was the poor woman to do? She designed the following scheme. She put three white balls and one black ball into an urn. Each daughter was to draw a ball from the urn at random without replacement on Thursday and not to reveal what color of ball she had drawn. On Friday morning, the next sets of suitors would come by. The daughters who had drawn white balls were instructed to slip notes under their doors saying yes to two of them and maybe to one of them on Saturday morning, and to agree to the marriage on Sunday. The daughter who had drawn a black ball was instructed to slip notes under their doors saying maybe to all of them on Saturday morning, and to refuse the marriage on Sunday. The mother made it common knowledge that this is what she had instructed the girls to do. It was well-known in the community that the girls strictly obeyed their mother.

| | |
|---|---|
| Thursday | Each daughter draws a ball from an urn with 3 white balls and 1 black ball without replacement. |
| Friday Morning | Each daughter receives a visit from a set of three suitors. |
| Saturday Morning | White-ball daughters distribute two yes's and one maybe;<br>The black-ball daughter distributes three maybes. |
| Sunday | White-ball daughter agrees to marry;<br>The black-ball daughter refuses to marry |

Table 1: Time table

If a suitor receives a maybe on Saturday morning, then what is the chance that he would be married on Sunday? Let M stand for the suitor being married, A for the suitor receiving a maybe, B for having asked the hand of a girl with a black ball and W for having asked the hand of a girl with a white ball. We calculate:

(1) $P(\text{M}|\text{A}) = P(\text{MB}|\text{A}) + P(\text{MW}|\text{A})$

(2) $P(\text{M}|\text{A}) = P(\text{M}|\text{BA})P(\text{B}|\text{A}) + P(\text{M}|\text{WA})P(\text{W}|\text{A})$

Since $P(\text{M}|\text{BA}) = 0$ and $P(\text{M}|\text{WA}) = 1$

(3) $P(\text{M}|\text{A}) = P(\text{W}|\text{A})$

(4) $$P(\text{M}|\text{A}) = \frac{P(\text{A} \mid \text{W})P(\text{W})}{P(\text{A} \mid \text{W})P(\text{W}) + P(\text{A} \mid \text{B})P(\text{B})}$$

(5) $$P(\text{M}|\text{A}) = \frac{(\frac{1}{3})(\frac{3}{4})}{(\frac{1}{3})(\frac{3}{4}) + 1(\frac{1}{4})} = 1/2$$

Hence, the chance of marrying upon receiving a maybe is 1/2.

Each of the girls received a set of three suitors the coming Friday. On Friday evening the mother visited one of the brothers in each set of suitors and offered them the following bet for three coins of gold: if the daughter they were courting would agree to marry on Sunday, then she would pay them four coins of gold and zero coins otherwise. Each set of brothers owned the family fortune in common and they had given each other the license to draw from the fortune if they saw the opportunity to buy or sell a bet that was not less than fair. So all the brothers who received a visit from the mother agreed to this fair bet, since they knew that the girls had randomly drawn from an urn containing three white balls and one black ball. On Saturday evening, after the girls had slipped all their notes under the doors, the mother let all of the brothers know that she was buying the following bet from the first brother in each set who would reach her doorstep: she would pay two coins on condition that she would receive four coins on Sunday if the girl they were courting would say agree to marry on Sunday and zero coins otherwise. Within each set there was at least one brother who was eager to bet, since at least one had received a maybe.

| | "Girl Agrees to Marry" | "Girls Refuses to Marry" |
|---|---|---|
| Bet 1, offered on Friday evening | Brother buys bet for 3 c<br>If "…", then<br>Mother pays prize of 4 c | Brother buys bet for 3 c<br>If "…", then<br>Mother pays prize of 0 c |
| Bet 2, offered on Saturday evening | Mother buys bet for 2 c<br>If "…", then<br>Brother pays prize of 4 c | Mother buys bet for 2 c<br>If "…", then<br>Brother pays prize of 0 c |
| Payoffs | Brothers lose 1 coin<br>Mother gains 1 coin | Brothers lose 1 coin<br>Mother gains 1 coin |

Table 2: Bets

For each daughter that agreed to marry on Sunday, the mother had a net gain of one coin: From each lucky set of suitors, she had taken in three coins and had to pay out four coins in the first bet, while she had paid out two coins and was taking in four coins in the second bet. For the daughter that refuses to marry on Sunday, she also had a net gain of one coin: From the unlucky set of suitors, she had taken in three coins and had to pay out zero coins in the first bet, while she had paid out two coins and was taking in zero coins in the second bet. Since there are three lucky sets of suitors and one unlucky set of suitors, she could count on a sure gain of four coins.

From this small fortune, she spent three coins on the dowries for the first three daughters to marry. The unlucky set of suitors came back the next week for the remaining daughter, and she was married the next Sunday with the remaining coin. They all lived happily ever after and, according to Thracian custom, had many, many children.

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