Indicative conditionals: the logic of assertion

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Abstract

I argue that indicative conditionals are best viewed as having *partial truth conditions*: "If A, B" is true if A and B are both true, false if A is true and B is false, and lacks truth value if A is false. The truth conditions are shown to explain a variety of important phenomena regarding indicative conditionals, including Adams' Thesis about the assertability conditions of conditionals, and how indicative conditionals embed in more complex constructions. In particular, the truth conditions are shown to provide the semantic basis for characterising several distinct *logics* of indicative conditionals, of which the *logic of assertion* is the main focus of the paper.

1 Introduction[1]

The aim of this paper is to provide an analysis of a particular aspect of logic of indicative conditionals—a topic rife with both controversy and confusion. The analysis builds on two theses. The first is that indicative conditionals 'express propositions' in the sense that they have truth conditions—albeit only partially—and embed in complex sentences. The second thesis is that there is not one but *three* relations that carry the kind of normative import that make them deserving of the epiteth 'logics'—one of these will be called 'the logic of assertion'—the topic of this paper. This logic is, I will show, is in some ways 'non-standard' in that it allows for semantically triggered *non-monotonicity*. The main task will be to show how the logic of assertion is related to the partial truth conditions of indicative conditionals and the truth conditions for the other standard sentential connectives.

On the first thesis the indicative conditional "If A, then B" has the same truth value as B, unless A is false in which case the conditional lacks truth value. The idea is not new (see in particular Quine (1950)) and is known to have some important virtues (see for instance McGee (1989), McDermott (1996), and Cantwell (2006a), but it is highly controversial—in part due to the lack of a good account that links the partial truth conditions of indicative

[1]Few can combine the trait of being a philosopher of the highest quality while seeing and appreciating what other philosophers are trying to do (and often enough explaining why they are not quite succeding in doing it). Combine this with warmth and a keen interest in people and one has Wlodek: it is a great honour to contribute to a volume celebrating his sixtieth birthday.

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conditionals to their *logic*. The second thesis in a sense explains the difficulties in finding such an account: there are three different kinds of logic involved and accounts of indicative conditionals that fail to take this into consideration and instead run the three logics into one, will invariably stumble on the seemingly incoherent multitude of semantical intuitions and the clash of data from what we know about our inferential practices.

Here are three distinct questions one can ask about a given pair of sentences A and B:

1. Is anyone who believes A committed to believing B?
2. Is anyone who makes the assumtion that A (in an argument) committed to accepting B?
3. Does one, whenever one asserts A, entitle the audience of the assertion to believe B (provided the audience understands what A and B mean)?

The questions are very different in character, but they all involve normative issues and what can losely be called the 'logical relationship' between the sentences A and B. As the three questions are different in character we should be open to the possibility that the answers to the question do not converge on a single logic and so we should, to be ready to err on the cautious side, distinguish between *the logic of belief*, *the logic of suppositional reasoning* and *the logic of assertion*. The distinction holds good even if the three logics turn out to be coextensive, for their intensions differ. As it happens I think one can make a strong case that the three different logics differ in extension as well as in intension. In Cantwell (2006a) I made the case for the distinction between the logic of belief and the logic of suppositional reasoning (in Section 4.3 below I restate the main ideas and results of that analysis), here the focus will be on the logic of assertion.

The difference between the logics can be explained by the difference of their subject matters. To believe that A is not the same as to reason under the assumption that A. For instance, one can legitimately *assume* that A even if one believes that A is false, and while we expect someone who believes that A to act as if A is true, we don't expect someone who in an argument assumes that A to also *act* as if A is true. On top of this, an assertion that A is clearly something very different from a belief or an assumption that A.

The basic source of normative constraints on the logic of assertion, I will argue, is that by asserting A one gives the audience to the assertion license to *believe* (or *accept*) A, as opposed to, say, giving the audience license to *suppose* or *assume* A. This will mean that the logic of assertion will draw on the logic of belief (rather than on the logic of suppositional reasoning). But, I will argue, the logic of assertion is distinguished from the logic of belief by the social character of assertion, in particular, by the fact that only a small subset of beliefs can be asserted and so the propositional context of an assertion differs systematically from the propositional context of a belief. This means that one can say 'too little': saying more provides the audience with more information about the epistemic context in which other assertions have been made and can serve to block inferences that would be legitimate in the absence of the extra information. This is, I will argue, the source of a particular kind of semantically characterizable non-monotonicity in the logic of assertion and it is closely related to the epistemic role of indicative conditionals (as tools for exploring possibilities that may not obtain).

The structure this paper is as follows. First I present some of the data—both linguistic and inferential—that an account of the semantics of indicative conditionals and the logic of assertion should explain. I then proceed to discuss the truth conditions of indicative conditionals and some problems of how these interact with the other truth-functional connectives. This is followed by a discussion of the 'belief-transplant' model for the normative basis of assertion which is contrasted with the 'assertion-as-premise-giving' model. The paper ends with a proposed semantical characterisation of the logic of assertion.

2 'The data'

2.1 Embedded conditionals

Indicative conditionals embed in complex constructions in various ways:

(1) Bill is coming and if Mary is coming as well there will at least be two guests.

(2) Either it is the case that if Mary is coming then Bill is coming, or it is the case that if Jane is coming then Oscar is coming (I can't remember which).

(3) It is not the case that if you bet on heads you won.

(4) There was a new apartment for sale and if Mary had enough money she bought it.

(5) Some snakes were so poisionous that everyone who was bitten died if he or she was not given an antidote.

(6) If my car has been stolen, then if Mary didn't steal it, Bill did.

(7) If every girl that was bitten survived if she was given the antidote, then the antidote works.

An account of indicative conditionals should cover how they contribute to the meaning of sentences in which they are embedded and it should tell under what conditions it makes sense to assert a sentence containing a (possibly embedded) indicative conditional.

Several writers[2] have held that conditionals do not 'really' embed, that constructions where it 'appears' that they do (as in (1)-(7) above) can be 'rewritten' as constructions that do not contain embedded conditionals using *ad hoc* principles. I think this explanatory strategy is doomed, conditionals embed too deeply and so systematically that any account of how they contribute to the meaning of a complex expression must be just as systematic as an account of how, say, 'and' contributes to the meaning of complex expressions—witness

[2]See in particular Gibbard (1981), Edgington (1995), Woods (1997) and Bennett (2004). Levi (1996) has also argued extensively against full embedability of conditionals..

(7) that contains a conditional embedded in the scope of a quantifier that in turn occurs in the antecedent of a conditional.

The sentence (7) also provides clear evidence of a kind of construction that has been held to be wholly absent in English or to demand extraordinary treatment: a conditional embedded in the antecedent of another conditional. Now, I agree that it is very difficult (if at all possible) to find meaningful constructions in English where *unquantified*—'naked'—conditionals occur in the antecedent of another conditional. Holding sentences like (7) in mind should, however, immunize us from drawing too extravagant conclusions about the semantics of conditionals from this linguistic fact.

The issue is confused by the existence of constructions like (the example is due to Gibbard 1981):

(8) If the cup broke if it was dropped, it was fragile.

Such sentences are standardly assumed to be of the form

(9) If [if the cup was dropped, then it broke], then it was fragile.

That is, sentences like (8) are standardly taken to be instances of naked conditionals occurring in the antecedent of another conditional. But whereas the surface syntactic form of (8) *may* be derived from (9) (I am not a grammarian and I know of no deeper study of the grammar of conditionals that establishes the grammatical relation between (8) and (9)), it is clear that its *semantic* interpretation is not. For (8) is equivalent to:

(10) If the cup was dropped and broke, it was fragile,

which in turn is equivalent to

(11) If the cup was dropped, then if it broke, it was fragile.

That is, the English construction "If A if B, C" should not be taken to be an instance of "If [If B, then A], then C", but rather to be an instance of "If B, then [if A, then C]". It is still an instance of an embedded conditional, but not an instance of a naked conditional embedded in the antecedent of another conditional.

2.2 'Standard' inferential patterns

An account of indicative conditionals should also get the logic right. For instance, it should explain why someone who asserts (1) commits himself to:

(12) If Mary is coming there will at least be two guests.

It should explain why (2) entails:

(13) If both Mary and Jane are coming then either Bill or Oscar is coming,

just as it should explain why (6) is taken to be equivalent to:

(14) If my car has been stolen and it wasn't Mary that stole it, then Bill stole it.

It should explain why when we combine (7) with

(15) Every girl that was bitten survived if she was given the antidote,

we can conclude "The antidote works".

2.3 'Non-standard' inferential patterns

2.3.1 The indicative conditional $\neq$ the material conditional

It seems clear that the assertion:

(16) Mary did it,

by truth functional means licenses the inference to:

(17) Either Mary or Bill did it,

But the assertion (16) does not license the inference to:

(18) If Mary didn't do it, Bill did.

For anyone who believes (16) is committed to believing (17), but you can believe (16) without also believing (18).

Consider instead the exchange:

– Either Mary or Bill did it.

– Yes, Mary did it.

Compare with the exchange:

– If Mary didn't do it, Bill did.

– No, if Mary didn't do it, Anne did.

One can imagine a situation where the first exchange would be just as appropriate as the second exchange. Say that I have evidence that points towards either Mary or Bill as the culprit while you have conclusive evidence showing that a female did it and that it most likely was Mary. In my state of belief I can utter the disjunction "Either Mary or Bill did it" which is perfectly consistent with what you believe, indeed you can fill me in and note that it is, in fact, Mary—no problem. Of course, I also believe the *conditional* "If Mary didn't do it, Bill did". But if I assert the conditional instead of asserting the disjunction, you will correct me, for you believe "If Mary didn't do it, Anne did" (Anne being the only remaining female candidate). So when I utter the disjunction you agree and fill in the missing information while when I utter the conditional, you disagree and correct me.

A central problem in an analysis of the indicative conditional is to account for the fact that while the indicative conditional "If not-A, B" is so close in meaning to the disjunction "Either A or B" they do not have the same logical force.

2.3.2 Transitivity?

Most agree that the inferential scheme of Transitivity (for conditionals) has a very strong *prima facie* claim to validity. For instance the assertion sequence

(19) If Anne came to the party, Jim came as well. If Jim came, Mary came as well,

licenses the inference to:

(20) If Anne came to the party, Mary came as well.

Yet at the same time one can find cases where it is reasonable to believe "If A, then B" and "If B, then C" while denying "If A, then C". For instance, say that you know that Mary is interested in Jim and that she will take the first opportunity to make a pass at him, but that she would never do it in the presence of Jim's wife Anne (indeed Mary would never go to a party that Anne attended). Say that you also know that Jim nearly always goes to parties alone as Anne doesn't like parties, but that on those few occasions where Anne does go to a party, Jim always goes along. Say, finally, that you know that Mary knew who was coming to the party. As you expect that if Jim went to the party he went alone, you accept "If Jim came, Mary came". You also accept "If Anne came to the party, Jim came", but you reject "If Anne came to the party, Mary came", just as you reject "If Jim and Anne came to the party, Mary came".

This is an example of a discrepancy between the logic of assertion (which validates transitivity) and the logic of belief (which doesn't). Ernest Adams (1975) who acknowledges the normative force of the inferential scheme of Transitivity in the logic of assertion but who also accepts the counterexamples in the logic of belief, tries to explain the discrepancy by suggesting that the assertion sequence:

(21) If A, then B. If B, then C,

is really *elliptical* for:

(22) If A, then B. If A and B, then C.

This would explain why the assertion sequence (21) licenses the inference to:

(23) If A, then C.

For the inference from (22) to (23) holds in Adams' logic of belief (as it should).

I think we can improve on Adams' account by giving a semantic explanation for why asserting (21) is equivalent to asserting (22), which explains why transitivity holds in the logic of assertion while allowing Transitivity to be violated in the logic of belief.

2.3.3 Restricted strengthening of the antecedent?

I think anyone who asserts (referring to a dinner party that one didn't attend)

(24) They had twelve chairs. If Anne came, they had just enough chairs,

licenses the inference to

(25) If Anne came, twelve chairs were just enough.

Again this marks a difference between the logic of belief and the logic of assertion. Say that I know that the hosts of the dinner had twelve chairs, and you believe twelve people were coming to the dinner and that Anne wasn't one of them. However, I believe that in the unlikely event that Anne came, she brought her own chair (bringing the total number of chairs up to thirteen). So I believe both of the claims in (24), but I do not believe in (25).

This an instance of the general scheme that the assertion sequence "A and if B, then C" licenses the inference to "If B, then A and C" (actually the scheme needs to be qualified somewhat to be valid, more on this below). That is, we allow the information that has been provided 'outside' of the conditional to 'carry over' into the consequent (and hence to the consequent) of the conditional. Again I think we can give a semantic explanation for this fact as well as an explanation for why the logic of assertion in this case differs from the logic of belief in this respect.

2.3.4 Modus ponens?

Two decades ago Vann McGee (1985) noted a puzzling fact:

> Opinion polls taken just before the 1980 election showed the Republican Ronald Reagan decisively ahead of the Democrat Jimmy Carter, with the other Republican in the race, John Anderson, a distant third. Those apprised of the poll results believed, with good reason:
>
> > If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.
> > A Republican will win the election.
>
> Yet they did not have reason to believe
>
> > If it's not Reagan who wins, it will be Anderson. (McGee, 1985, p.462)

The first moral to be drawn from McGee's story is that *modus ponens* does not hold universally in the context of belief: one can believe both "If A then if B then C" and "A" without thereby being committed to believing "If B then C".

But an equally (or perhaps even more) puzzling fact escaped McGee's attention. Assume that someone with the above beliefs asserted:

(26) A Republican will win the election. If a Republican wins the election, then if it's not Reagan it will be Anderson.

I think an audience to this assertion (that did not know the background story) could justifiably infer:

(27) If it's not Reagan who wins the election it will be Anderson.

That is, the logic of assertion is closed under modus ponens while the logic of belief is not.

2.3.5 Non-Monotonicity!

In McGee's story it was reasonable to believe both that Reagan, and hence a Republican, won and to believe that if Reagan didn't win, Carter did. Having these two beliefs you might assert:

(28) A Republican won. If Reagan didn't win, Carter did.

When speaking to an audience that lacks knowledge about Carter's party-affilitation, such an assertion would license the audience to infer:

(29) If Reagan didn't win, then Carter was a Republican.

This is a problem as you (the speaker) doen't believe (29). That is, by asserting only things that you believe (as is the case in (28), you can license an inference to a claim that you don't believe. The problem with asserting (28) (and no more) is that you have thereby said *too little*.

There is a simple way of blocking the problematic inference: add the information that Carter wasn't a Republican. The assertion sequence

(30) A Republican won and Carter was not a Republican, but if Reagan didn't win, Carter did,

provides *more* information than (28) but blocks the inference to (29). This means that the logic of assertion is *non-monotonic*. Indeed I think that this is a semantically triggered non-monotonicity that a semantics of conditionals should explain.

3 Truth conditions

3.1 Truth conditions vs. Assertability conditions

Much of the literature on indicative conditionals have focused on their *conditions of assertability*, taking *Adams' Thesis* (Adams 1975) as a starting point:

(AT) The *degree of assertability* of a conditional $A \to B$, goes by the conditional probability of B *given* A, i.e. the higher the subjective probability $\Pr(B|A)$, the more assertable the conditional $A \to B$.

I think there is an important element of truth to Adams' thesis, but I also think that it is a big mistake to take the thesis as the primary starting point in an analysis of the meaning or semantics of indicative conditionals. For assertability conditions tell us only half the story, they tell us what it takes for a speaker to be justified in making a claim. The other untold half of the story is what it takes for a claim to be *right* or *wrong*, we need the conditions under which we the audience can *vindicate* or *impugne* a claim by acknowledging that the speaker was right or wrong.[3]

You can be justified in asserting the conditional "If Jim went to the party, Mary went as well" even though you do not know whether Jim, or Mary, or anyone else, went to the party. This does not mean that the question whether Jim went to the party, or whether Mary went to party, is irrelevant when it comes to deciding whether your claim was correct. No matter how well justified your assertion (maybe you justifiably but mistakenly believed that Jim and Mary are twins attached at the hip), it will face the tribunal of the impunging "You were wrong!" if it turns out that Jim went to the party and Mary didn't. A one-sided focus on assertability conditions will miss the intersubjective element that transcends the doxastic state of the speaker at the time of the utterance (that transcends the question whether the speaker was justified in making the assertion at the time of the utterance) in favour of the question whether the claim was correct or not, whether the facts bare it out or not. This intersubjective element can, I think, be captured most succinctly in terms of *truth conditions*.

We know the conditions under which your assertion "If Jim went to the party, Mary went as well" is vindicated or impugned and how the vindication/impugnment of an assertion relates to the truth value of the sentence (or proposition) asserted. Your assertion "If Jim went to the party, Mary went as well" is vindicated—the conditional asserted is *true*—if it turns out that both Jim and Mary went to the party. Your assertion is impugned—the conditional asserted is *false*—if it turns out that Jim went but Mary didn't. If it turns out that Jim didn't go, your claim is neither vindicated or impugned; if it turns out that Jim didn't go, we exclaim neither "You were right!" nor "You were wrong!"; if it turns out that Jim didn't go, the conditional lacks truth value.

In general (letting $A \to B$ stand for the indicative conditional "If A, then B"):

$A \to B$ is *true* if and only if B is true and A is not false.

$A \to B$ is *false* if and only if B is false and A is not false.

One might expect that the assertability conditions for a sentence is, or should be, a derivative of the sentence's conditions of vindication:

> One is justified in asserting A *if and only if* one is justified in beliving that the conditions under which the assertion that A would be vindicated are or will be fulfilled.

But this equivalence does hold in general. For instance it is widely held that one does *not* conform to the assertability conditions for the disjunction $A \vee B$ ("A or B") if one (i)

[3] I owe this use of 'vindicating' and 'impunging' assertions to Belnap (2001).

already believes that A and (ii) one's sole reason for believing $A \vee B$ is that one believes that A. The favoured explanation is given by Grice: by asserting $A \vee B$ rather than A one would violate the norm that prescribes that one should be as informative as possible. Still, even though the assertability conditions for asserting $A \vee B$ are not satisfied, an assertion of $A \vee B$ will be vindicated if it turns out that A is true. Thus one can believe that an assertion will be vindicated without being justified in making the assertion.

The converse direction also fails: one can be justified in making an assertion even though one does not believe that the assertion will be vindicated. For you can be justified in asserting "If Jim went to the party, Mary went as well" even though the possibility that Jim didn't go to the party is consistent with your beliefs. And so—as your claim can be vindicated only if Jim went to the party—it is consistent with your beliefs that the conditions under which your claim would be vindicated do not and will not obtain. Putting it more directly and in a form that might be seen as more provocative: you can be justified in asserting "If Jim went to the party, Mary went as well" even though you do not believe that "If Jim went to the party, Mary went as well" is true, for the conditional is true only if Jim went to the party, and you can be justified in asserting the conditional even though you do not believe that Jim went to the party.

Your belief "If Jim went to the party, Mary went as well" does not commit you to the belief that the conditional in question is true, it commits you only to the belief that it is true *if it has a truth value*. This turns out to be the key to understanding Adams' thesis. For the probability of $A \to B$ becomes the probability that $A \to B$ is true *given* that it has a truth value, i.e. the probability of $A \wedge B$ (the conditional is true only if the antecedent and consequent are both true) divided by the probability of A (the conditional has a truth value only if A is true), i.e. $\Pr(A \to B) = \Pr(A \wedge B)/\Pr(A) = \Pr(B|A)$.[4]

Belief contravening conditionals—conditionals where you believe the antecedent to be false—are of particular interest. Say that you believe that Jim didn't go to the party. Then you believe of the conditional "If Jim went to the party, then Mary didn't" that it will be neither be vindicated or impugned just as you believe of the conditional "If Jim went to the party, Mary went as well" that it will be neither vindicated or impugned. What needs to be explained is the asymmetry that you can be justified in believing and asserting one of the conditionals even though you are not justified in believing or asserting the other; how there can be an asymmetry in a situation where you believe of both conditionals that it will neither be vindicated nor impugned, where you believe both conditionals to lack truth value.

There is no great mystery involved. You know that Jim and Mary never leave each other's sides, furthermore Jim told you that he wasn't going to the party, so you have quite justifiably concluded that neither Jim nor Mary went to the party. But you are still justified in believing that if Jim went, Mary went as well. Of course, if Jim went to

[4]I am assuming here that A does not itself contain a conditional, when it does, the $A \to B$ can be true even if A lacks truth value.

the party then your belief that he didn't go is false, thus your belief in the conditional "If Jim went to the party, Mary went as well" hinges on what you believe to be the case if your belief that Jim didn't go to the party is false. And you believe that even if you are wrong about Jim not going to the party (he might have changed his mind), that does not make you wrong about the fact that Jim and Mary never leave each other's sides. So you believe that if the conditional "If Jim went to the party, Mary went as well" has a truth value (which you believe it doesn't), then it is *true*. And you also believe that if the conditional "If Jim went to the party, then Mary didn't" has a truth value (which you believe it doesn't), then it is *false*.

Belief contravening conditionals are useful in exploring what I will call *secondary scenarios*, in exploring what is the case if some of your beliefs (such as the belief that Jim didn't go to the party) are wrong. They are also, I think, the source of much of the 'erratic' behaviour of conditionals in the logic of assertion. But more on this below.

3.2 Truth conditions vs. content

There is a legitimate and, I think, important sense in which the indicative conditional $A \to B$ has the same *informational* or *propositional content* as its counterpart the *material implication* $\neg A \lor B$ ("not-A or B"). Your assertion "If Jim went to the party, Mary went as well" tells me no more and no less about Jim, Mary and the party they might have gone to than your assertion "Either Jim didn't go to the party, or Jim and Mary both went". In both cases I will, if I come to accept what you have asserted, come to reject the possibility that Jim went and Mary didn't. The claim $A \to B$ rejects precisely the same possibilities as the claim $\neg A \lor B$, for $A \to B$ is *false* if and only if $\neg A \lor B$ is false, both claims reject the possibility that A is true and B false.

In general: two propositions A and B have the *same* informational content when A is false is every assignment where B is false and vice versa. I think it makes sense to tie the notion of informational content to the *falsity* conditions of a proposition rather rather than to its *truth* conditions. At the very least there is an asymmetry between falsity and truth conditions that tends to be overlooked in purely bivalent settings where falsity and truth are interdefinable. For when one asserts A one is asking the audience to the assertion to *reject* every possibility ('possible world') in which A is false, but one is not in any comparable sense also asking the audience to *endorse* every possibility ('possible world') in which A is true. Indeed a standard crude but effective measure of the *amount* of information conveyed by a claim is how many possibilities ('possible worlds') the claim rejects.

The close relationship between $A \to B$ and $\neg A \lor B$ is mirrored in the following equivalence: if one does not believe A to be false, then one is commited to believing the conditional $A \to B$ if and only if one is commited to believing the corresponding disjunction $\neg A \lor B$. Here is a crude derivation of this claim. First, we know that anyone who believes $A \to B$ is commited to believing $\neg A \lor B$. Second, say that one believes $\neg A \lor B$ and that one does not believe $\neg A$. Then the possibility that $A \land B$ ("A and B") is consistent with ones beliefs while the possibility that $A \land \neg B$ is not. Thus one believes that if $A \to B$ has a truth value—which it has only if A is true—then B is true, and so one believes that if $A \to B$ has a truth value, then it is true. So if one believes that $\neg A \vee B$ and one does not believe $\neg A$, then one is commited to believing $A \to B$.

Keep in mind, however, that this 'belief equivalence' only holds under specific constraints, for if one believes that A is false then one can believe $\neg A \vee B$ without being commited to believing $A \to B$. Keep in mind also that the vindication conditions of $A \to B$ and $\neg A \vee B$ differ: the assertion "Either Jim didn't go to the party, or Jim and Mary both went" would be vindicated if Jim didn't go to the party, while the corresponding "If Jim went to the party, Mary went as well" would neither be vindicated or impugned under the same conditions. This difference is reflected in a difference in truth conditions: $\neg A \vee B$ is true if A is false, but under the same condition $A \to B$ lacks truth value. The material $\neg A \vee B$ shares its falsity conditions with $A \to B$, but not its truth conditions.

The major insight of thinkers like Paul Grice (1989), Frank Jackson (1979, 1987) and David Lewis (1976, 1986), was to see that while the assertability conditions of the indicative and material conditional differ, their contents coincide. The big mistake of Grice, et al., was to conclude that thereby the truth conditions of the indicative and material conditionals coincide and that the difference between $A \to B$ and $\neg A \vee B$ resides solely in their assertion conditions. This was a mistake as the differences in the *assertion* conditions of $A \to B$ and $\neg A \vee B$ do not explain the difference in the *vindication* conditions of $A \to B$ and $\neg A \vee B$. And the latter difference gives rise to a difference in the *truth* conditions of $A \to B$ and $\neg A \vee B$ which means that the difference between $A \to B$ and $\neg A \vee B$ is semantic and not only pragmatic.

3.3 Compund sentences

The development of the non-bivalent analysis for indicative conditionals has been seriously hampered by problems in the truth functional analysis of compound constructions. On the analysis I will propose expressions containing *multiple conflicting scenarios* come with slightly different truth conditions than expressions that are free of such conflicts.

The natural extension of the truth conditions for conjunction to deal with the possibility that some claims lack truth value is:

$A \wedge B$ is *true* if and only if A is true and B is true.

$A \wedge B$ is *false* if and only if A is false or B is false.

According to this analysis $A \wedge (B \to C)$ lacks truth value if A is true and B is false (as $B \to C$ then lacks truth value which means that neither the truth nor the falsity condition for conjunction is satisfied).

McDermott (1996) has noted that these truth conditions seem to fly in the face of some of our linguistic intuitions. Discussing bets on the outcome of a die, McDermott considers:

(31) It will be an even number and if its above three it will be a four.

McDermott reports that all his subjects agree that a bet on (31) is won if the die lands on a four and that it is lost if it lands on an uneven number or a six. Some subjects hold that a bet on (31) is *canceled* if the die lands on a two. This is consistent with the truth conditions given for $\wedge$ above, for according to those truth conditions (31) lacks truth value if the die lands on a two. However, McDermott also reports that some subjects hold that a bet on (31) is *won* if the die lands on a two, which (according to McDermott) suggests that according to these subjects a conjunction like (31) is *true* when one conjunct is true and the other lacks truth value. This, of course, goes contrary to the suggested truth conditions for $\wedge$ above.

Summarizing the data McDermott suggests that the English 'and' is *ambiguous*, it sometimes can be taken to have the truth conditions of $\wedge$ and sometimes the truth conditions of $\cap$:

> $A \cap B$ is *true* if and only if neither A nor B is false and at least one of A or B is true.
>
> $A \cap B$ is *false* if and only if A is false or B is false.

With these alternate truth conditions a bet on (31) would be won if the die shows a two.

McDermott is understandably reticient about postulating an hitherto undiscovered ambiguity in the most common word in the English language and he tries to lessen the impact of the ambiguity thesis. The ambiguity involved is not, we are told, of the same kind as the ambiguity of, say, 'bank'. Rather:

> It is the kind of ambiguity that arises when a concept that is unified in ordinary applications permits of two natural extensions in cases of a rarer kind. (p.15)

I share the ambivalence that McDermott reports about a sentence like (31). On the one hand there is a clear sense in which only the first half of sentence (31) has been put to the test if the die shows a two: someone who placed a bet on "If it's above three it will be a four" has clearly *not* won the bet if the die lands on a two. This is a *prima facie* case for holding that (31) lacks truth value when the die shows a two. On the other hand there is a clear sense in which *part* of the claim (31) has been vindicated, as the claim "The die will show an even number" is a non-trivial truth-carrying consequence of (31) that has been vindicated, indeed *every* truth-carrying consequence of (31) has thereby been vindicated. Thus as one is right about everything that one can be right about, one could hold that the claim as a whole has been vindicated and hence that (31) is true.

However, I do not think that McDermott's ambiguity thesis is the best explanation for this phenomenon. Rather I think the heart of the problem is that the conceptual resources at our disposal when we discuss whether a claim is right or not is too rich to be reduced to truth conditions. If I assert "Jim will come to the party and so will Mary" there is a sense in which I am 'half-right' if Jim comes to the party and Mary doesn't, even though the sentence asserted is just plain false under these conditions (indeed I am inclined to think that I am *only* 'half-wrong' under these conditions even though the sentence I asserted is clearly false). Similarly, if I assert a universal quantification "All X's are Y's" I will often be held to be 'nearly right' if practically all X's are Y's even though an occasional X may not be a Y. There is nothing mysterious about such 'half-truths', but it is clear that it is part of a richer discourse than we capture by truth conditions.

I think we should rest content with the position that (31) is *strictly speaking not true* (nor false) when the die lands on a two, but that it still conveyed non-trivial true information (indeed, that it is 'half right'). The ambivalence that one can hold towards (31) comes from the fact that while one is only 'half right' when the die lands a two, one is not in any corresponding sense 'half wrong', for the second conjunct isn't false, it merely lacks truth value.

The above comments are by no means conclusive. While $A \wedge B$ and $A \cap B$ are not in general logically equivalent, they always have the same propositional content (one is false if and only if the other is false) which makes them too close semantically, I think, for our pre-theoretical intuitions to serve as reliable arbitrators when we decide which of $\wedge$ or $\cap$ serves as the best interpretation of the English 'and'. To settle the issue we must adopt a wider theoretical perspective, in particular I think a decisive factor is the role that the truth conditions play in the basis for logic. My favouring $\wedge$ before $\cap$ is ultimately justified by (i) the semantic basis for determining what follows from the *sequence* of assertions $A_1, \ldots, A_n$, and (ii) the hypothesis that such a sequence of assertions $A_1, \ldots, A_n$ is always equivalent to an assertion of the conjunction A_1 *and* $\cdots$ *and* A_n. But more on this below.

3.4 Multiple Scenario Conflicts

McGee (1989) notes that (pertaining to the flip of a coin) a construction like:

(32) If it doesn't land heads, it will land tails and if it doesn't land tails, it will land heads,

cannot be true (for if the coin lands heads the first conjunct lacks truth value, and if the coin lands tails the second conjunct lacks truth value) but it can be false (if the coin lands on its edge)—at least as long as we interpret 'and' as $\wedge$. Thus according to the above truth conditions the only reasonable degree of belief one can have in (32) is zero (a bet on (32) can be lost but it cannot be won) and this seems highly counterintuitive.

McDermott cites McGee's example as speaking in favour of the ambiguity thesis. If one interprets the 'and' in (32) along the truth conditions of $\cap$ rather than $\wedge$, (32) turns out to be logically equivalent to

(33) Either the coin will land heads or it will land tails,

and this clearly satisfies our pre-theoretical intuition that (32) says no more and no less than (33).

I agree with McDermott that the 'and' in (32) should be interpreted along the lines of $\cap$ rather than $\wedge$, but I maintain that this does not support the ambiguity thesis as ambiguity arises only when there are at least two reasonable interpretations and in this case there is only one: $\cap$. Rather, I attribute the phenomenon to a semantically triggered shift in the truth conditions for 'and', the 'semantic trigger' being that the conjuncts are in a *(semantic) multiple scenario conflict*.

> A non-empty set of sentences Γ are in a *(semantic) multiple scenario conflict* if and only if (1) there is no assignment that simultaneously satisfies (makes true) every member of Γ, and (2) there is at least one assignment where no member of Γ is false.

For instance, your beliefs “If Jim went to the party, Mary went as well” and “Jim didn’t go to the party” are in a multiple scenario conflict: both are perfectly legitimate beliefs, and it is perfectly legitimate to hold both beliefs simultaneously, but there is no way in which both beliefs can be true (if Jim didn’t go to the party, the conditional “If Jim went to the party, Mary went as well” lacks truth value, while if Jim went to the party the unconditional “Jim didn’t go to the party” is false). The categorical belief “Jim didn’t go to the party” pertains to your *primary scenario*, while the belief contravening conditional “If Jim went to the party, Mary went as well” explores a *secondary* scenario which is in conflict with your primary scenario. The point of being able to explore a secondary scenario is, of course, that you may be wrong about the primary scenario, maybe Jim went to the party after all.

Note that multiple scenario conflicts are not logical contradictions, indeed, according to the definition a mulitple scenario conflict arises only when there is some assignment of truth values where no proposition in the conflict set is false.

McGee’s example (32) is another form of multiple scenario conflict. Here the claims “If the coin doesn’t land heads...” and “If the coin doesn’t land tails...” explore different scenarios (in one scenario the coin doesn’t land heads, in the other it doesn’t land tails) which we can see by noting that it is not possible for both conjuncts to be true, but it is possible for one conjunct to be true while the other lacks truth value. In this case the conflict is not between a ‘primary’ scenario and a ‘secondary’ scenario, for neither of the conditionals are belief contravening, one might say that they explore different possible primary scenarios.

Yet another example of a multiple scenario conflict is:

(34) If Jim went to the party, Mary went as well. If he didn’t go, then neither did she.

On one scenario, Jim went to the party, on the other, he didn’t; each conditional explores a different possibility, if one conditional is true, the other lacks truth value, and vice versa.

Now, the hypothesis is that the English ‘and’ is not ambiguous, but that its truth conditions depend on whether the conjuncts involved are in multiple scenario conflict or not. I suggest that we introduce a third connective $\sqcap$ with truth conditions that coincide with $\wedge$ as long as the conjuncts are free of conflict, and with $\cap$ as soon as the conjuncts are in conflict. What complicates things is that $\sqcap$ is thus not a purely truth functional connective, we cannot tell from knowing only the truth values of A and B, whether $A \sqcap B$ is true or nor not, we need to know also if the two are in conflict or not. For instance in $(A \to B) \sqcap (A \to C) \sqcap (\neg B \vee \neg C) \sqcap D$ no two conjuncts are in conflict by themselves but $(A \to B)$, $(A \to C)$, and $\neg B \vee \neg C$ are jointly in conflict while D does not participate in any conflicts at all. We want truth conditions for $\sqcap$ that are not sensitive to the order of the conjuncts.

$A_1 \sqcap \cdots \sqcap A_n$ is *true* if and only if (i) no A_i is false, and (ii) for any A_i that lacks truth value, there is a non-empty conflict-free subset Γ of $\{A_1, \ldots, A_n\}$ where at least one element A_j of Γ is true and the set-theoretical join of Γ and $\{A_i\}$ is in conflict.

$A_1 \sqcap \cdots \sqcap A_n$ is *false* if and only if some A_i is false.

For the two-conjunct case:

Observation 1

- *If A and B are free of conflict, then $A \sqcap B$ is logically equivalent to $A \wedge B$.*
- *If A and B are in multiple scenario conflict, then $A \sqcap B$ is logically equivalent to $A \cap B$.*

For the multi-conjunct case, note that

$$(A \rightarrow B) \sqcap (A \rightarrow C) \sqcap (\neg B \vee \neg C) \sqcap D$$

becomes logically equivalent to

$$(A \rightarrow (B \sqcap C)) \sqcap (\neg B \vee \neg C) \sqcap D$$

which is true if and only if $\neg B \vee \neg C$, D and $\neg A$ is true, and false if and only if one of $\neg B \vee \neg C$, D or $\neg A$ is false.

McDermott holds not only that 'and' is ambiguous, but also that the English 'or' is ambiguous between $\vee$ and $\cup$:

$A \vee B$ is *true* if and only if A is true or B is true.

$A \vee B$ is *false* if and only if A is false and B is false.

$A \cup B$ is *true* if and only if A is true or B is true.

$A \cup B$ is *false* if and only if either A is false and B is not true, or B is false and A is not true.

He adduces examples like (again referring to the outcome of a toss of a die):

(35) It will either be a one, or if its above three it will be a six.

(36) If it above three it will be a six, or if its below three it will be a one.

According to McDermott the 'or' in (35) is ambiguous between $\vee$ and $\cup$ while the 'or' in (36) should be interpreted as $\cup$. In particular, he notes that when the die lands on a two one can be inclined both towards holding that (35) is false and towards holding that it lacks truth value, while in the same situation (36) is to be considered false.

I think McDermott here is over-inspired by the symmetry between 'and' and 'or', in particular, I think McDermott makes too much of the fact that $\cap$ relates to $\cup$ in the same way that $\wedge$ relates to $\vee$—given 'conditional negation' (see below) they are interdefinable. But the analogy is far from complete, for while $A \wedge B$ and $A \cap B$ have the same propositional content, $A \vee B$ and $A \cup B$ do not. Indeed letting $\cup$ capture the truth conditions of 'or' in any situation at all leads to very strange consequences. For it implies that if one asserts "If its above three it will be a six" then one will not be impugned if the die lands a two, but if one asserts the seemingly *weaker* "Either it will be a one, *or* if its above three it will be a six" then one will be impugned if the die lands a two. In effect, according to the $\cup$ interpretation, a disjunction can be stronger (reject more possibilities) than one of its disjuncts.

I do not think the examples produced by McDermott provide a convincing case that the English 'or' is ever interpreted as $\cup$. In particular I think McDermott mistakenly reads the 'or' in (36) as an 'and', i.e. McDermott interprets (36) as he would interpret:

(37) If it above three it will be a six, *and* if its below three it will be a one.

Note that (37) becomes false if the die lands on a two. I think we bring out the contrast between (36) and (37) better if we rephrase the former somewhat:

(38) Either it is the case that if its above three it will be a six, or it is the case that if its below three it will be a one (I can't remember which/I am not permitted to say more).

There seem to be no grounds whatsoever for holding that such a claim is false or impugned if the die lands on a two.

There are basically two different ways of defining negation. *Inner* or *conditional* negation is defined:

$\neg A$ is true if and only if A is false.

$\neg A$ is false if and only if A is true.

It corresponds to the following kind of denial:

– If Jim went, then so did Anne.

– No: if Jim went, Anne didn't go.

For according to inner or conditional negation, $\neg(J \rightarrow A)$ is equivalent to $J \rightarrow \neg A$.

A stronger form of negation is *outer* negation:

$\neg A$ is true if and only if A is false.

$\neg A$ is false if and only if A is not false.

It corresponds to the following kind of denial:

– If Jim went, then so did Anne.

– No: Jim went and Anne didn't.

According to outer negation $\neg(J \rightarrow A)$ is equivalent to $J \wedge \neg A$.

Below I will assume that $\neg$ is interpreted as inner or conditional negation as this seems to be the way it behaves in most constructions of the form "It is not the case that if A, B", but it may be that English uses both forms of negation.

4 The normative basis of the logic of assertion

The aim of this paper is to characterize the 'logic of assertion', that is the relation between a sequence of assertions $A_1, \ldots, A_n$ and what an audience to that assertion sequence is entitled to accept on the basis of the assertion sequence (in what follows, let $A_1, \ldots, A_n \mid\!\sim B$ be a short-hand for "The assertion sequence $A_1, \ldots, A_n$ entitles the audience to accept B").[5] So the aim is to provide a descriptively adequate characterisation of a relation that is inherently normative. The success of the project depends crucially upon identifying the normative basis for the logic of assertion, to identify what it is about assertion that introduces a normative dimension?

I think the 'belief-transplant' model of assertion is the key. It can be summarized by the three principles:

A-E By Asserting A one Entitles the audience to believe that A.

E-C If one Entitles the audience to believe that A, then one is Committed to believing that A.

E-C-E If one Entitles the audience to believe that A, and anyone who believes that A is Committed to believing that B, then one Entitles the audience to believe that B.

In brief: an assertion is the speaker's attempt to turn one of his or her beliefs into a belief shared by the audience to the assertion.

This is still a rough sketch. There are three reasons to be more pedantic about the precise structure of the belief-transplant model. First, what is the underlying model of

[5] I will use the term 'assertion sequence' even though I will not be concerned with those aspects of assertion sequences (such as anaphoric relations) where the order of what has been asserted is important—I could just as well use the term 'assertion set'. Also, I am interested primarily in the semantically characterizable aspects of the logic of assertion, thus I will not be interested in pragmatically justified inference such as the fact that the assertion that A licenses the audience to the assertion to infer that the speaker believes that A.

belief? (Highly relevant when embarking on the descriptive enterprise of characterising the logic.) Second, there is a naive version of the belief transplant model of assertion that gets things wrong, we need to see why. Third, there is an alternative to the belief transplant model—the 'assertion-as-premise-giving'-model—that gets things wrong as well, again we need to see why.

4.1 The model of belief

There are (at least) three different ways to view belief in the context of assertion. One way is to think of belief as 'high subjective probability' which places the kind of belief one needs to make an assertion within a wider context of action (assuming that ones degrees of belief, or subjective probabilities, are intimately related to how one is disposed to act). A second way is to equate belief with the kind of thing one has (or pretends to have) when one makes or assents to an assertion. A third way is to equate belief with what is taken for granted or assumed (though not only for the sake of the argument) in enquiry and action, the kind of thing that can serve as a *detacheable* premise in the logic of suppositional reasoning.

I think there is a point to each of these models and that neither has any priority to being the one that characterises our 'ordinary' use of the term 'belief'. Indeed they may not even be competing models. In particular I think probabilists are inclined to think that the categorical term 'belief' corresponds to high subjective probability and that high subjective probability is both necessary and sufficient for a claim to be assertable. The probabilist would then consider the third model of belief as a special case: the case where 'high' subjective probability is 'certainty'—probability 1.

Now, I will adopt the probabilist model of belief even though I think it is a less than perfect model for belief in the context of assertion. For instance, I think it is an empirical fact that a sequence of assertions $A_1, \ldots, A_n$ brings about the same commitments as an assertion of the conjunction $A_1 \sqcap \cdots \sqcap A_n$. That is, the following principle of Sequence-Conjunction Equivalence holds:

S-C-E $A_1 \sqcap \cdots \sqcap A_n \mathrel{|\!\!\sim} B$ if and only if $A_1, \ldots, A_n \mathrel{|\!\!\sim} B$

Any probabilistic model of belief must in some way qualify the relationship between assertibility and 'high' subjective probability. We know (witness the 'lottery paradox') that it is possible to consider each of a set of propositions $A_1, \ldots, A_n$ to be highly probable while at the same time considering the conjunction $A_1 \sqcap \cdots \sqcap A_n$ to have zero probability. Thus it would seem that one cannot assert just any sequence of highly probable propositions (as this would violate S-C-E). A related problem is to explain why it seems inappropriate to assert, say, "Lottery ticket 251 will not win" (and no more) when all one knows about the lottery is that there are ten thousand lottery tickets and that only one ticket can win.

The main methodological bonus of choosing the probabilistic model of belief is that it provides means for exploring the logic of belief that does not presuppose an already existing characterisation of the logic of assertion. Of course, we must also be aware that in a non-bivalent setting the standard laws of probability do not hold. The 'betting interpretation' of subjective probability allows us to derive the new set of laws of non-bivalent probability (see Cantwell 2006b):

For any truth-determinate sentences (= sentences that cannot lack truth value) A and B:

1. $0 \leq \Pr(A) \leq 1$.
2. If A and B are logically equivalent, then $\Pr(A) = \Pr(B)$.
3. $\Pr(\neg A) = 1 - \Pr(A)$.
4. $\Pr(A \vee B) = \Pr(A) + \Pr(B)$, if A and B are mutually inconsistent.

For every sentence A:[6]

5. $\Pr(A) = \Pr(Tr(A))/\Pr(TV(A))$, if $\Pr(TV(A)) > 0$.

Using (5) we can show that for any truth-determinate A and B:

$$\begin{aligned}\Pr(A \to B) &= \Pr(Tr(A \to B))/\Pr(TV(A \to B)) \\ &= \Pr(A \wedge B)/\Pr(A).\end{aligned}$$

If the conditional probability $\Pr(B|A)$ is given the standard betting interpretation whereby $\Pr(B|A)$ corresponds to the betting quotient of a bet on B conditional upon A (i.e. a bet on B that is canceled if and only if A is false), it follows in addition that $\Pr(A \to B) = \Pr(B|A)$.[7] That is, given the partial truth conditions for the indicative conditional and the laws of non-bivalent of probability we can *derive* Adams' Thesis instead of postulating it.

4.2 The single scenario is the default

The claim that an assertion entitles the audience to assertion to believe what has been asserted should be handled with some care. A belief always exists in a richer structure of beliefs and dependencies among beliefs. For instance my belief "Reagan will win the election" (recalling Section 2.3.4) together with my belief "Reagan is a Republican", are the reasons I have for believing "A Republican will win the election". While a belief automatically exists in such a richer structure of beliefs and dependencies (in virtue of being a belief), an assertion—which is public—is not automatically interpreted within the same structure of dependencies.

For instance if I merely assert

(39) A Republican will win the election. If a Republican wins the election and it's not Reagan it will be Anderson,

[6]The operator $Tr(A)$ ("it is true that A") has the following truth conditions: $Tr(A)$ is true if and only if A is true, $Tr(A)$ is false if and only if A is not true. The operator $TV(A)$ ("A has a truth value") has the following truth conditions: $TV(A)$ is true if and only if A is either true or false, $TV(A)$ is false if A lacks truth value.

[7]Note that due to the fact that $\to$ is non-bivalent and so as the standard laws of probability do not hold in an unrestricted way, the non-bivalent laws of probability do not collapse in the way described in Lewis' triviality result (see Lewis (1976)).

I have expressed two of my beliefs, but I have not expressed or reported the crucial conflict between the beliefs. I have not expressed or reported that for me the conditional "If a Republican wins the election and it's not Reagan it will be Anderson" is belief contravening (I believe that its antecedent is false).

In this case a simple version of the Belief Transplant picture of the logic of assertion misses something important. Plucking out a belief and presenting it to an audience by means of an assertion is to strip the belief of the web of belief that surrounds it and presenting it naked: the result is a radical change of context. This difference in context is, I think, the fundamental circumstance that drives a wedge between the logic of belief and the logic of assertion. It is the key difference between the subjective notion of a belief and the social notion of an assertion.

My failure to report, or indicate, the conflict between my belief "A Republican will win the election" and my belief "If a Republican wins the election and it's not Reagan it will be Anderson", has a direct result: contrary to my mandate I have entitled you (the audience) to infer "If Reagan doesn't win the election, Anderson will". That is, I have entitled you to believe something that I myself do not believe—I did not provide a 'closed' assertion sequence:

> A set of propositions $\{A_1, \ldots, A_n\}$ is *closed* (for the speaker) if for every B such that $A_1, \ldots, A_n \mathrel{|\!\sim} B$, the speaker is commited to believing B.

I venture that an audience is entitled to the default assumption that an assertion sequence is closed and that it is the responsibility of the speaker to make sure that it is. As a result the audience will be entitled to take everything that a speaker asserts as either belonging to the same primary scenario, or to be possible qualifications to the primary scenario, until such an interpretative strategy becomes impossible due to their being a multiple scenario conflict in the asserted material.

This default assumption explains a particular kind of non-monotonic behaviour among indicative conditionals. Recall the example of Section 2.3.5:

28. A Republican will win the election. If it isn't Reagan who wins, it will be Carter.

30. A Republican will win the election, and Carter isn't a Republican, but if it isn't Reagan who wins, it will be Carter,

With (28), but not with (30), one justifies the inference to:

29. If Reagan didn't win, then Carter was a Republican.

Thus even though one asserts more in (30) than in (28) there are inference that (28) justifies that (30) doesn't. And the reason is that (30) contains an explicit multiple-scenario conflict which (28) doesn't, and so by default (28) is taken to speak of one and the same scenario.

4.3 Assertion-As-Premise-Giving

There is a distinct rival to the Belief Transplant view. According to Robert Brandom

> [t]he function of assertion is making sentences available for use as *premises in inferences*. (Brandom (1994) p.168, my italics)

Some care is needed when stating the view. To begin with adopting a premise in reasoning is not something for which we need permission or need even justify: any claim, no matter how far-fetched, can function as a premise in further reasoning. It seems reasonable to interpret Brandom as meaning that the function of assertion is making sentence available for use as *detacheable* premises in inferences. That is, by asserting that A the speaker licenses the audience to *believe* that A and to believe anything that follows when taking A as a premise in further reasoning. The crucial question is: what kind of reasoning?

In Cantwell (2006b) I argued that the logic of *suppositional reasoning* is just classical logic or, depending on how one interprets negation, some close relative to classical logic. The derivation, in brief, goes as follows. Let S be a *suppositional state*—a set of sentences that are either *believed*, *accepted* or *assumed* to hold. Suppositional states are assumed to be closed under their commitments, so if accepting $A_1, \ldots, A_n$ commits one to accepting B, then every suppositional state containing $A_1, \ldots, A_n$ contains B. The minimal static commitments assumed are the commitments that arise from:

(EQ) If A and B are logically equivalent then anyone who accepts A is commited to accepting B.

(CS) If B is true in every assignment where A is not false, then anyone who accepts A is commited to accepting B.

The rationale for these constraints is given by the following result:

Theorem 2 (Cantwell 2006b)
Let A and B be propositions. The following two claims are equivalent:

1. *For any non-bivalent probability measure Pr: $Pr(A) \leq Pr(B)$.*

2. *A and B are logically equivalent (have the same truth value in every assignment) or B is true in every assignment where A is not false.*

That is, (EQ) and (CS) give the precise semantic constraints for guaranteeing that a suppositional state is 'closed upwards' under probability.

The normative dynamics of suppositional reasoning is driven by the possibility of adding new premises. Let $S * A$ stand for the result of supposing that A when in the suppositional state S. The main constraints are the *Ramsey Test*[8] and *Success*:

[8] The Ramsey Test is so-called after the famous footnote (Ramsey 1929, p.155): "If two people are arguing If A will B? and are both in doubt as to A, they are adding A hypothetically to their stock of knowledge and arguing on that basis about B...".

(RT) $B \in S * A$ if and only if $A \to B \in S$.

(Success) $A \in S * A$.

Now define the relation $\vdash$:

> $A_1, \ldots, A_n \vdash B$ if and only if $B \in S * A_1 \cdots * A_n$, for every suppositional state S.

Theorem 3 (Cantwell 2006a)
$\vdash$ is precisely classical logic (when negation is interpreted as 'outer' negation).

Thus classical logic (or a close relative) summarises the constraints shared by *all* suppositional states. We also have the following result:

Theorem 4 (Cantwell 2006a)
$A_1, \ldots, A_n \vdash B$ if and only if in every assignment where none of $A_1, \ldots, A_n$ are false, B is not false.

So the logic of suppositional reasoning (which is classical logic if negation is interpreted as 'outer' negation) preserves 'non-falsity', i.e. the logic of suppositional reasoning is 'content preserving'.

These results suggest that if an assertion that A entitles the audience to believe A and everything that follows on the supposition that A, the logic of assertion just becomes classical logic. I think this is deeply implausible as it would mean that my assertion "Mary is going to the party" would entitle the audience to accept "If Mary isn't going to the party, she is going to run a marathon in under thirty-five minutes" as this inference is classically valid.

But for several reasons these results cannot immediately be translated into Brandom's framework. In Brandom's inferentialist programme one's inferential commitments go far beyond the logic of the 'logical constants' ('and', 'or', etc.). Conceptual truths bring about the same kind of inferential commitments, for instance, the assumption that Gothenburg is south of Stockholm commits one to accepting that Stockholm is north of Gothenburg. Indeed I share with Brandom the view that this is a valid inference, even though it is not validated by classical logic. But Brandom goes still further, for in his account background beliefs also shape the inferential commitments. So for instance I, when reasoning under the assumption that I will go to work in the morning, am commited to accepting that I will go there by the subway, as this is how I usually go to work. This inferential commitment can be overruled by the additional assumption that I will take a taxi, making my inferential commitments non-monotonic—a crucial feature of Brandom's account.

Still, Brandom's views about inferential commitments seem to share some basic structural similarities with the logic of suppositional reasoning as described above. In particular Brandom's view that the indicative conditional is the linguistic tool that 'makes explicit' one's inferential commitments, can be interpreted as an analogoue of the Ramsey Test, or, in a different format, of the Deduction Principle in the logic of assertion:

(DP) $\Delta \mid\!\sim A \to B$ if and only if $\Delta, A \mid\!\sim B$.

However, I do not think that (DP) is valid in the logic of assertion. In particular, I do not think that the right-to-left direction (Conditional Proof) holds:

(CP) If $\Delta, A \mid\!\sim B$, then $\Delta \mid\!\sim A \to B$.

For, coupled with a very weak monotonicity assumption:

(VWM) $A, B \mid\!\sim A$,

we get a property (Unrestricted Qualification) that I think we should reject in the logic of assertion:

(UQ) $A \mid\!\sim B \to A$.

Of course, many instances of it seem to make sense at a glance. For instance, if I assert:

(40) The last person to leave the room wore red socks,

you might apply conditional proof to conclude:

(41) If Anne was the last person to leave the room she wore red socks.

Now, the inference from (40) to (41) isn't all that bad (after all: the inference is validated by classical logic), but the question is: did I entitle you to believe (41) on the basis of my assertion (40)?

Turn it around. Say that I believe that Anne wasn't the last person to leave the room, and that I am absolutely certain that she wasn't wearing red socks. Then I could very well believe (40), deny (41), and instead believe:

(42) If Anne was the last person to leave the room, the last person to leave the room wasn't wearing red socks.

So *my* belief in (40) does not commit me to believe (41), hence, if I by asserting (40) entitle you to believe (41), I entitle you to believe something that I do not even though I believe what I asserted. Something must be wrong.

One response is that this just shows that my assertion sequence (40) wasn't *closed*—that I should have said more. To avoid confusing the audience I should have added that Anne wasn't the last person to leave the room. After all, the 'say more to mean less' strategy is the strategy I have suggested for other cases where one says too little and implies too much. In the present case, however, that strategy just wont work. For according to Unrestricted Qualification $B \to A$ follows from A, *for every* B. Thus to make sure that you do not inadvertently infer $B \to A$ from my assertion A, I would have to explicitly disawow every B for which I believe $B \to \neg A$ and that might be a very long list: communication under such circumstances would be painfully tedious. Thus Unrestricted Qualification, as well as Conditional Proof, have to go.

This said, I think there is a sense in which one by asserting A entitles the audience to treat A as a premise of suppositional reasoning. The entitlement, however, is not an entitlement for the audience to *believe*, it is an entitlement to *question*. Brandom puts it well:

> In asserting a sentence, one not only licenses further assertions (for others and for oneself) but commits oneself to *justifying* the original claim... Specifically, in making a claim, one undertakes the conditional task responsibility to demonstrate one's entitlement to the claim, if that entitlement is brought into question. (Brandom 1994, p.172)

Consider the exchange (wherein Adam knows that the Prince and Princess were to get married yesterday, but without having finally verified that this is so) :

Adam: The prince and princess got married yesterday.

Sophie: Oh yeah? Even if the princess was run over by a truck?

Adam: Well, maybe not then.

Sophie: What if she found out that her father is dying?

Adam: Then the wedding was probably postponed.

Sophie: And if the princess found out that the prince has been unfaithful?

Adam: She didn't.

Sophie: But if she did?

Adam: Then it was canceled, but she didn't.

Sophie: What if the princess found out that the prince has an incurable illness that will cause rashes all over his body and cause his limbs to fall off?

Adam: She loves him so much that she wouldn't care.

The exchange has the following structure: First Adam makes a claim (A). Sophie then queries whether $B \to A$ (that is, whether A will hold even if B) for various B's. Adam has a number of different possible replies depending on the B in question.

(i) He can reject B. In the above exchange this is what happens when he denies that the princess found out that the prince has been unfaithful. By rejecting B and asserting $B \to \neg A$, Adam makes it clear that he is ready to stick to the unconditional claim A even though he rejects $B \to A$. I do not think that he is thereby withdrawing his previous claim, as his assertion A never licensed Sophie to believe $B \to A$ to begin with (and note that he still maintains that A).

(ii) Adam can reply "A even if B". This is what happened when he concluded that the princess loved the prince so much that she married him even if he had a ghastly illness. I do not think that he is acknowledging an already existing license for Sophie to believe $B \to A$, instead he is strengthening the initial claim: not only is he commiting himself to the claim that the princess married the prince, he is also commiting himself that she did it even if she found out that he had an illness.

(iii) Adam might be forced to withdraw from the unconditional A, perhaps offering the weaker "A if $\neg B$". This is what happened when Sophie raised the possibility that the princess had been run over by a truck or that the princess' father had died. In this case Adam is clearly withdrawing or qualifying his initial claim, but he isn't withdrawing from "A if B"—that the princess would get married even if the she was run over by a truck—for this he never licensed Sophie to believe, rather, he is withdrawing from "A"—the princess might not, after all, have gotten married yesterday.

Adam's original claim A did not entitle Sophie to believe each of $B \to A$, but it did give her the right to pose the questions of the form "A even if B?" and Adam is required (within reason) to respond. Sophie's questioning functions as a way of probing the strength of Adam's initial claim and what qualifications the initial claim can handle. Treating a claim as a *premise* is to treat it as invulnerable to qualifications (as $B \to A$ follows on the supposition that A for arbitrary B). When Adam claims that the princess didn't find out that the prince has been unfaithful (case (ii)) he is not claiming that the princess married the prince even if she found out, so he concedes that his initial claim is vulnerable to a conceivable qualification (another one would be:"What if the universe exploded yesterday?", "It didn't."), but he is also denying the relevance of this qualification by denying that the qualifying circumstance occurred (the princess didn't find out that the prince has been unfaithful, the universe didn't explode yesterday).

So, if the above argument is correct, the indicative conditional does not satisfy the Deduction Principle in the logic of assertion. This does not mean, however, that we should reject Brandom's idea that the indicative conditional functions to make explicit our inferential commitments. But that idea does not automically translate into the Deduction Principle. We see this best if we state Conditional Proof in the English form that comes closest to capturing the idea that indicative conditional function to make explicit our inferential commitments. On this reading Conditional Proof claims that we can move from "If I were to assert A that would commit me to B" to "I am commited to $A \to B$". But as the above argument illustrates, this move fails in those circumstances where one believes A to be false. For instance, I would not assert that Anne was the last person to leave the room as I believe that she wasn't, still if I were to make this assertion, I would be commited (via my earlier assertion (40)) to the claim that Anne wore red socks. But this counterfactual relationship does not make me commited to (41), to the claim that if Anne was the last person to leave the room she wore red socks.

5 The semantic basis for the logic of assertion

5.1 Single Scenario Logic

I think the logic of assertion, when restricted to the non-conflicting single scenario case, can be characterised semantically by:

SSL When $\{A_1, \ldots, A_n\}$ contains no conflict:

$A_1, \ldots, A_n \mid\!\sim B$ if and only if

1. in every assignment of truth values where all of $A_1, \ldots, A_n$ are true, B is true (Truth Preservation),

2. in any assignment where B is false, *some* sentence A_i is false (Falsity Inheritance).

One reason for thinking that this is so is the following result:

THEOREM 5
Let A and B be propositions. The following two claims are equivalent:

1. $A \mid\!\sim B$.
2. *There is some probability measure Pr such that $Pr(Tr(A)) > 0$ and for every non-bivalent probability measure Pr: if $Pr(Tr(A)) > 0$, then $Pr(A) \leq Pr(B)$.*

(For proof, see appendix.)

That is, SSL corresponds to that part of the logic of belief that governs propositions that one does not believe lack truth value. This is, of course, precisely what we want as the default assumption of single scenario logic is that the asserted material belongs to the same scenario, i.e. is simultaneously satisfiable.

McDermott (1996) proposed an unrestricted version of SSL, a version where the premises $A_1, \ldots, A_n$ need not be free of conflict, but the unrestricted version gets things wrong. For instance, the inference from A and $\neg A \to B$ to $\neg A \to C$ satisfies both Truth Preservation (trivially! there is no model in which both the premises are true) and Falsity Inheritance (if $\neg A \to C$ is false, then A is false), but the inference $A, \neg A \to B \mid\!\sim \neg A \to C$ should no more be part of the logic of assertion than the inference $A \mid\!\sim \neg A \to C$.

The logic (SSL) can be characterized by a number of inference rules. In the following assume that $\Gamma \cup \{A, B\}$ and Δ are conflict-free sets of sentences (this implies that A and B are simultaneously satisfiable).

Structural conditions:

(Mon) If $\Gamma \mid\!\sim D$ and $\Gamma \subseteq \Gamma'$, then $\Gamma' \mid\!\sim D$, if Γ' is not in conflict.

(Reflexivity) $A \mid\!\sim A$

(Cut) If $\Gamma \mid\!\sim D$ and $\Delta \mid\!\sim E$ for each $E \in \Gamma$, then $\Delta \mid\!\sim D$

Axioms and rules for standard connectives:

($\sqcap I$) : $A, B \mid\!\sim A \sqcap B$.

($\sqcap E$) : (i) $A \sqcap B \mid\!\sim A$, (ii) $A \sqcap B \mid\!\sim B$.

($\vee I$) : (i) $A \mid\!\sim A \vee C$, (ii) $A \mid\!\sim C \vee A$.

($\vee E$) : If $\Gamma, A \mid\!\sim D$ and $\Gamma, B \mid\!\sim D$, then $\Gamma, A \vee B \mid\!\sim D$.

($\neg I$) If $\Gamma, A \mid\!\sim \perp$, then $\Gamma \mid\!\sim \neg A$

($\neg E$) If $\Gamma, \neg A \mid\!\sim \perp$, then $\Gamma \mid\!\sim A$

($\perp I$) $A, \neg A \mid\!\sim \perp$, if A contains no occurrence of a conditional (note that when $\neg$ is interpreted as inner negation $\neg(A \to B)$ is equivalent to $A \to \neg B$ which does not logically contradict $A \to B$).

So $\mid\!\sim$ behaves 'classically' with respect to the standard non-conditional connectives. It also satisfies a restricted form of monotonicity: as long as new premises are not in conflict with the old premises, everything that could be inferred from the old premises can be inferred from the old premises together with the new premises.

The indicative conditional does *not*, however, behave classically. While we have:

(Modus Ponens) $A, A \to B \mid\!\sim B$, (if $A \to B$ is true then B is true, hence Truth Preservation is satisfied; say that B is false, if A is false then Falsity Inheritance is immediate, if A is not false, then $A \to B$ is false, and so again Falsity Inheritance is satisfied), we do not have Conditional Proof. For instance, we have $A, B \mid\!\sim A$ as soon as A and B are not in conflict, but we do not in general have $A \mid\!\sim B \to A$ (say that A is true and B is false, then $B \to A$ lacks truth value and so Truth Preservation is violated).

Instead of Conditional Proof we have a variety of principles beginning with a Weak Introduction rule:

(WI) $A, B \mid\!\sim A \to B$,

which allows the audience to infer "Mary is coming even if Jim is coming" '(*Jim is coming* $\to$ *Mary is coming*) from the assertion sequence "Mary is coming. Jim is coming.".

Transitivity holds:

(Transitivity) $A \to B, B \to C \mid\!\sim A \to C$,

For assume that $A \to B$ and $B \to C$ are both true, then B and C are both true while A is not false, and so $A \to C$ is true which means that Truth Preservation is satisfied. Assume that $A \to C$ is false, then C is false and A is not false; if B is not false then $B \to C$ is false, while if B is false, $A \to B$ is false, so Falsity Inheritance is satisfied. Just to avoid confusion, let me stress that while Transitivity holds in the logic of assertion, it doesn't hold in the logic of belief: one can believe (= consider highly probable) $A \to B$ and $B \to C$ without believing $A \to C$.

The single scenario assumption becomes most obvious in the properties of Restricted Antecedent Strengthening and Restricted Consequent Strengthening (recall that we are assuming that A, B and C are not in conflict):

(RAS) $A, B \to C \mid\!\sim (A \sqcap B) \to C$.

(RCS) $A, B \to C \mid\!\sim B \to (A \sqcap C)$.

Assume that A and $B \to C$ are both true, then A and C are true and B is not false, then $A \sqcap B$ is not false and $A \sqcap C$ is true so $(A \sqcap B) \to C$ is true as is $B \to (A \sqcap C)$ which means that (RAS) and (RCS) both satisfy Truth Preservation. Assume that $(A \sqcap B) \to C$ is false, then C is false and $A \sqcap B$ is not false, i.e. B is not false, but then $B \to C$ is false, so (RAS) satisfies Falsity Inheritance. Assume that $B \to (A \sqcap C)$ is false, then either A is false or C is false. In the first case Falsity Inheritance for (RCS) is satisfied, in the second case, note that B is not false, so $B \to C$ is false and again (RCS) satisfies Falsity Inheritance.

Both principles (RAS) and (RCS) make explicit the idea that a conditional (in this case $B \to C$) is by default to be regarded as a possible qualification of the primary scenario (here given by A) rather than as presenting a competing secondary scenario.

Restricted Antecedent Strengthening should *not* be confused with its *un*restricted sibling:

$$B \to C \mid\!\sim (A \sqcap B) \to C,$$

which isn't validated by the semantics (assume that B and C are both true and that A is false, then $B \rightarrow C$ is true but $(A \sqcap B) \rightarrow C$ lacks truth value as the antecedent is false).

We have:

$$A \rightarrow B \mathrel{|\!\sim} \neg A \vee B.$$

For assume that $A \rightarrow B$ is true. It follows immediately that B is true and, so, that $\neg A \vee B$ is true. Assume that $\neg A \vee B$ is false. Then A is true and B is false, i.e. $A \rightarrow B$ is false.

We do *not*, however, have the converse direction.

$$A \vee B \mathrel{|\!\not\sim} \neg A \rightarrow B,$$

for say that A is true, then $A \vee B$ is true, but then $\neg A \rightarrow B$ lacks truth value, violating truth preservation.

Finally we have a number of structural properties:

(→EQ) If $A \mathrel{\sim\!|\!|\!\sim} B$, then $A \rightarrow C \mathrel{|\!\sim} B \rightarrow C$

(→⊓) $A \rightarrow (B \sqcap C) \mathrel{\sim\!|\!|\!\sim} (A \rightarrow B) \sqcap (A \rightarrow C)$

(→∨) $A \rightarrow (B \vee C) \mathrel{\sim\!|\!|\!\sim} (A \rightarrow B) \vee (A \rightarrow C)$

(→¬) $A \rightarrow \neg B \mathrel{\sim\!|\!|\!\sim} \neg(A \rightarrow B)$

(→→) $A \rightarrow (B \rightarrow C) \mathrel{\sim\!|\!|\!\sim} (A \sqcap B) \rightarrow C$

5.2 Multiple Scenario Logic

Turn now to the case where an assertion sequence can contain a conflict. For reasons to be discussed below I do not think that the logic for such assertion sequences can be fully characterized by semantic means, but the semantics still gives valuable guidance.

Consider the following suggestion:

(MSL) $A_1, \ldots, A_n \mathrel{|\!\sim} B$ if and only if (i) B is true in every assignment of truth values I where for each j $(1 \leq j \leq n)$ either (a) A_j is true in I or (b) A_j lacks truth value in I and there is some non-empty conflict free subset Γ of $\{A_1, \ldots, A_n\}$ such that some element of Γ is true and the set theoretical union of Γ and A_j is in conflict, and (ii) if B is false in some assignment, then some A_j is false in that assignment.

This semantic definition of the multiple scenario logic is a mouthful and as opposed to the single scenario logic I have no direct justification for the semantic characterisation of the logic. Rather, it has been chosen to yield the following result:

Theorem 6
$A_1, \ldots, A_n \mathrel{|\!\sim} B$ if and only if $A_1 \sqcap \cdots \sqcap A_n \mathrel{|\!\sim} B$. (For proof, see appendix.)

The first thing to note is that $\mathrel{|\!\sim}$ now has become a non-monotonic logic. For instance we have (for logically independent A, B and C):

$$A, B \rightarrow C \mathrel{|\!\sim} (A \sqcap B) \rightarrow C,$$

but we do *not* have:

$$A, B \to C, \neg B \mid\!\sim (A \sqcap B) \to C,$$

(let $\neg B$ and A be true, then $B \to C$ is not false but is in conflict with $\neg B$, yet $(A \sqcap B) \to C$ is not true, violating condition (i) of (MSL)) nor do we have

$$A, B \to C, B \to \neg A \mid\!\sim (A \sqcap B) \to C.$$

So while the inference from:

(43) A Republican won, Reagan didn't win $\to$ Carter did

to

(44) Reagan didn't win $\to$ Carter is a Republican,

is valid, the inference from

(45) A Republican won, Carter is not a Republican, Reagan didn't win $\to$ Carter did

to

(46) Reagan didn't win $\to$ Carter is a Republican,

is not.

Nevertheless, we still have have a property of Weak Monotonicity:

(WM) If $\Gamma \mid\!\sim C$ and $\Delta \mid\!\sim D$, then $\Gamma \cup \Delta \mid\!\sim C \vee D$.

Unconditional sentences always pertain to the primary scenario, so for such sentences we still have Monotonicity (Restricted to unconditional sentences):

(RM) If C is a sentence containing no conditionals and $\Gamma \mid\!\sim C$, then $\Gamma \cup \Delta \mid\!\sim C$.

So, for instance, the inference from

(47) Reagan won, Reagan was a Republican, Reagan didn't win$\to$ Carter did,

to

(48) A Republican won,

is valid even though the premises in the inference are in conflict.

As already noted Modus Ponens is valid when the premises contain no conflicts. So from

(26) A Republican won the election. If a Republican won the election, then if it wasn't Reagan it was Anderson,

one can infer

(27) If Reagan didn't win, Anderson did.

But when there are conflicts in the assertion set Modus Ponens no longer holds (reinstating McGee's conclusion that Modus Ponens is "not strictly valid" (McGee, 1985, p.462). The inference from

(49) Reagan won the election. Reagan is a Republican. If a Republican won the election, then if it wasn't Reagan it was Anderson,

to (27) is *not* valid. For while we can infer that a Republican won the election we cannot infer that if it wasn't Reagan who won, it was Anderson. We do *not* have $A, \neg B, A \to (B \to C) \mid\!\sim B \to C$. For take an assignment I where A and $\neg B$ are true. In that assignment $A \to (B \to C)$ lacks truth value but it is in conflict with $\neg B$ which is in the assertion set and is also true. As $B \to C$ also lacks truth value in I condition (ib) of MSL has been violated so $A, \neg B, A \to (B \to C) \not\mid\!\sim B \to C$.

All of this is as it should be, I think, but the Multiple Scenario Logic contains some obvious lacunae. For instance, we do *not* have

$$A \to B, \neg A \to C \mid\!\sim A \to B.$$

For take an assignment where A is false and C is true. In that assignment $A \to B$ lacks truth value, but $A \to B$ is in conflict with $\neg A \to C$ which is true in the assignment.

So (MSL) violates the eminently plausible principle of Minimal Monotony:

(MM) $A_1, \ldots, A_n \mid\!\sim A_i$, for each i such that $1 \leq i \leq n$.

This, I think most would agree, is not acceptable: the semantic characterisation given by (MSL) does not properly capture the full logic of assertion. However the failure is not quite as dramatic as it may seem, for on the level of *content*, (MSL) captures the right inferences. That is, we still have:

$$A \to B, \neg A \to C \mid\!\sim \neg A \vee B.$$

Indeed, we also have:

$$A \to B, \neg A \to C \mid\!\sim (A \sqcap B) \vee (\neg A \sqcap C)$$

In general, we find that as soon as the assertion set contains conflicts the multiple scenario logic defined by (MSL) does not give us all the inferences we have the right to expect from the logic of assertion. So we must conclude that there is more (primarily the principle (MM)) to the full logic of assertion than can be given by the semantics alone. I think the basic problem is that when we are dealing with assertion sequences containing conflicts the semantic considerations are too varied and complex for a straightforward semantical explication. It should be possible, however, to strengthen the basic (MSL) logic by adding non-semantical constraints such as (MM). I will not, however, explore the properties of the resulting logic here.

6 Concluding remarks

The truth conditions for the indicative conditional flow from the same considerations and theoretical prejudices that allow for the standard truth conditions of, say, disjunction and conjunction. There is thus nothing mysterious about letting indicative conditionals carry truth values. What makes indicative conditionals special is that they can lack truth value. This complicates their logic, not so much because it forces us to consider a 'three-valued' logic, but because the fact that indicative conditionals can lack truth value is what enables one of their main functions: to speak of, and reason with, the possibility that one is wrong about what I have called the 'primary scenario'.

A complicating factor is that our understanding of indicative conditionals is informed not only by linguistic data, but also by our use of indicative conditionals in reasoning and in ascribing and expressing beliefs about the world. The key to understanding indicative conditionals is to see that these represent very different kinds of data. Here and elsewhere I have argued that we should be very careful not confuse the *belief* that something is the case with the *assumption* that it is: assumptions are linked to conditionals via the Ramsey Test, beliefs are not. In this paper I have tried to show that even though the normative basis of the logic of assertion stems from the fact that an assertion entitles the audience to believe what has been asserted, the logic of assertion gets its distinctive non-monotonic mark from the fact that it is part of a *social* practice rather than a purely epistemic and subjective affair: not everything that one believes is publicly available, and even if one can make any particular belief publicly available, one cannot (due to practical considerations of time) make them all public.

The account that has been presented combines the virtues of Adams' Thesis with a truth functional account that allows for full embeddability of conditionals and a full semantic characterisation of an important fragment of the logic of assertion. Still, it should be clear that much work remains. Apart from problems already encountered in the analysis (in particular, finding a proper characterisation of the logic of multiple scenario conflicts), one would want to extend the analysis to a wider linguistic context: conditionals interacting with quantifiers, anaphoric relations, and so on.

Appendix: Proofs of Theorems

Proof of Theorem 5: Assume that $A \mid\!\sim B$. It follows that the singleton set $\{A\}$ is not in conflict, i.e. A can be true, i.e. there is some probability measure Pr such that $\Pr(Tr(A)) > 0$. Take any probability measure Pr such that $\Pr(Tr(A)) > 0$. It follows that $\Pr(TV(A)) > 0$, so by Law 5, $\Pr(A) = \Pr(Tr(A))/\Pr(TV(A))$. By truth-preservation $\Pr(Tr(A)) \leq \Pr(Tr(B))$ and by falsity-inheritance $\Pr(F(B)) \leq \Pr(F(A))$ (where $F(A)$ is true if and only if A is false, and $F(A)$ is false if and only if A is not false). As $TV(A)$ is truth determinate we have, by law 4, $\Pr(A) = \Pr(Tr(A))/(\Pr(Tr(A)) + \Pr(F(A)))$. As $\Pr(Tr(A)) \leq \Pr(Tr(B))$, $\Pr(Tr(A))/(\Pr(Tr(A))+\Pr(F(A))) \leq \Pr(Tr(B))/(\Pr(Tr(B))+ \Pr(F(A)))$ and as $\Pr(F(B)) \leq \Pr(F(A))$, $\Pr(Tr(B))/(\Pr(Tr(B)) + \Pr(F(B))) = \Pr(B)$

(by law 5). So $\Pr(A) \leq \Pr(B)$.

Assume that there is some probability measure Pr such that $\Pr(Tr(A)) > 0$ and that for every non-bivalent probability measure Pr: if $\Pr(Tr(A)) > 0$, then $\Pr(A) \leq \Pr(B)$.

(i) Truth-Preservation. Assume for reductio that there is some assignment I where A is true and B is not true. Define, for any truth-determinate C:

$$\Pr^*(C) = \begin{cases} 1, & \text{if } C \text{ is true at } I \\ 0, & \text{otherwise.} \end{cases}$$

$\Pr^*$ can be extended to a full probability measure Pr by (for any C):

$$\Pr(C) = \begin{cases} \Pr^*(Tr(C))/\Pr^*(TV(C)), & \text{if } \Pr^*(TV(C)) > 0, \\ 0, & \text{otherwise.} \end{cases}$$

We know that $\Pr(Tr(B)) = 0$ and either $\Pr(TV(B)) > 0$ in which case, by Law 5, $\Pr{*}(B) = 0$ or $\Pr * (TV(B)) = 0$. In either case $\Pr^*(B) = 0$. As $\Pr(A) = 1$ and as $\Pr(Tr(A)) > 0$ this contradicts our initial assumption.

(ii) Falsity-Inheritance. Assume for reductio that there is some assignment I where B is false and A is not false. As (by assumption) there is some probability measure $\Pr'$ such that $\Pr'(Tr(A)) > 0$ there is some assignment I' at which A is true. Define, for any truth-determinate C:

$$\Pr^*(C) = \begin{cases} 1, & \text{if } C \text{ is true at both } I \text{ and } I' \\ .5, & \text{if } C \text{ is true in precisely one of } I \text{ or } I' \\ 0, & \text{otherwise.} \end{cases}$$

$\Pr^*$ can be extended to a full probability measure Pr by (for any C):

$$\Pr(C) = \begin{cases} \Pr^*(Tr(C))/\Pr^*(TV(C)), & \text{if } \Pr^*(TV(C)) > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Now either (a) A is true at both I and I' in which case $\Pr^*(A) = 1$ or, (b) A is true at I' but lacks truth value at I, in which case, by Law 5, $\Pr(A) = \Pr(Tr(A))/\Pr(TV(A)) = .5/.5 = 1$. As $\Pr(TV(B)) > 0$ and as $\Pr(F(B)) > 0$, $\Pr(B) < 1$. But this contradicts our initial assumption.

□

Proof of Theorem 6: Assume that $A_1, \ldots, A_n \mid\!\sim B$. Take any assignment V where $A_1 \sqcap \cdots \sqcap A_n$ is true. It follows (from the truth definition for $\sqcap$) no A_j is false in V and that each A_j is either true in V or there is some non-empty conflict free subset Γ of $\{A_1, \ldots, A_n\}$ such that some element of Γ is true and the set theoretical union of Γ and A_j is in conflict. But then, as $A_1, \ldots, A_n \mid\!\sim B$, B is true in V as well.

Take any assignment where B is false. It follows (as $A_1, \ldots, A_n \mid\!\sim B$) that some A_j is false. But then $A_1 \sqcap \cdots \sqcap A_n \mid\!\sim B$.

Assume that $A_1 \sqcap \cdots \sqcap A_n \mid\!\sim B$. It follows that B is true in every assignment where $A_1 \sqcap \cdots \sqcap A_n$ is true and that $A_1 \sqcap \cdots \sqcap A_n$ is false in every assignment where B is false. Take any assignment V where each A_j is either true in V or there is some non-empty conflict free subset Γ of $\{A_1, \ldots, A_n\}$ such that some element of Γ is true and the set theoretical union of Γ and A_j is in conflict. It follows (by the truth definition of $\sqcap$) that $A_1 \sqcap \cdots \sqcap A_n$ is true in V and, so, that B is true in V.

Assume that B is false in some assignment, then $A_1 \sqcap \cdots \sqcap A_n$ is false and so some A_j is false in that assignment. Hence $A_1, \ldots, A_n \mid\!\sim B$.

$\square$

References

Adams, E. W. (1975). *The logic of conditionals*. Reidel, Dordrecht.

Belnap, N., Perloff, M., and Xu, M. (2001). *Facing the future*. Oxford University Press.

Bennett, J. F. (2003). *A philosophical guide to conditionals*. Oxford University press, Oxford.

Brandom, R. (1994). *Making it explicit*. Harvard University Press, Cambridge.

Cantwell, J. (2006a). Coping with non-bivalent conditionals. In Lagerlund, H., Lindström, S., and Sliwinski, R., editors, *Modality Matters*, pages 77–109. Uppsala Philosophical Studies.

Cantwell, J. (2006b). The laws of non-bivalent probability. *Logic and Logical Philosophy*, 15:163–171.

Edgington, D. (1995). On conditionals. *Mind*, 104:235–329.

Gibbard, A. (1981). Two recent theories of conditionals. In Harper, Stalnaker, and Pearce, editors, *Ifs*, pages 211–247. Reidel, Dordrecht.

Grice, H. P. (1989). *Studies in the way of words*. Harvard University Press, Cambridge MA.

Jackson, F. (1979). On assertion and indicative conditionals. *Philosophical Review*, 88:565–589.

Jackson, F. (1987). *Conditionals*. Basil Blackwell, Oxford.

Levi, I. (1996). *For the sake of the argument*. Cambridge University Press, Cambridge.

Lewis, D. (1976). Probabilities of conditionals and conditional probabilities. *Philosophical Review*, 85:297–315.

Lewis, D. (1986). Probabilities of conditionals and conditional probabilities II. *Philosophical Review*, pages 581–589.

McDermott, M. (1996). On the truth conditions of certain 'if'-sentences. *Philosophical Review*, 105:1–37.

McGee, V. (1985). A counterexample to modus ponens. *Journal of Philosophy*, 82:462–471.

McGee, V. (1989). Conditional probabilities and compounds of conditionals. *Philosophical Review*, 98:485–542.

Quine, W. V. O. (1950). *Methods of logic*. Holt, Rheinhart and Winston, New York.

Woods, M. (1997). *Conditionals*. Oxford University Press.