SUPERIORITY IN VALUE AND THE REPUGNANT CONCLUSION*

Karsten Klint Jensen

Danish Centre for Bioethics and Risk Assessment

kkj@foi.dk

ABSTRACT: James Griffin has considered a weak form of superiority in value a possible remedy to the Repugnant Conclusion. In this paper, I demonstrate that, in a context where value is additive, this weaker form collapses into a stronger form of superiority. And in a context where value is non-additive, weak superiority does not amount to a radical value difference at all. I then spell out the consequences of these results for different interpretations of Griffin's suggestion regarding population ethics. None of them comes out *very* successful, but perhaps they nevertheless retain some interest.

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It is a great pleasure to honour Wlodek. He is among the philosophers who have been most important for me. He is not only a brilliant philosopher. He also has the quality – rare among philosophers I am sorry to say – of being a very nice and generous person. To the benefit of us all, this characteristic transpires extremely fruitfully in the way he works with and discusses philosophy.

1. Introduction

John Stuart Mill famously introduced the notion of *superiority of quality* of a pleasure, claiming that[1]

> [i]f one of the two [pleasures] is, by those who are competently acquainted with both, placed so far above the other that they prefer it, even though knowing it to be attended with a greater amount of discontent, and would not resign it for any quantity of the other pleasure which their nature is capable of, we are justified in ascribing to the preferred enjoyment a superiority in quality, so far outweighing quantity as to render it, in comparison, of small account.

In recent decades, such superiority relations between different objects of value have been the subject of interest, probably because James Griffin,

[1] J. S. Mill: *Utiltarianism* (1861), quoted from *Utilitarianism, On Liberty, Considerations on Representative Government*, London: Dent, 1993, p. 9.

Derek Parfit and others have considered them a possible remedy to the Repugnant Conclusion in population ethics.

Parfit states different versions of the Repugnant Conclusion,[2] but what they seem to have in common is this: Suppose we have a scale of welfare. Consider some number of people *n* all living on a very high level ***a***. For any positive level of welfare, ***z***, however low, a population of *m* people at ***z*** is better than *n* people at ***a***, provided *m* is large enough. Parfit and many others consider this conclusion repugnant. It follows straightforwardly from the Utilitarian Total Principle (if $m\boldsymbol{z} > n\boldsymbol{a}$ and hence if $m > n\boldsymbol{a}/\boldsymbol{z}$). But according to Parfit, it follows from any reasonable principle of beneficence, provided that *mere addition* of people at a positive level of welfare does not make an outcome worse and that the principle 'if ***y*** is not worse than ***x*** and ***z*** is better then ***y***, then ***z*** is better than ***x***' holds for betterness.[3]

In a note, Griffin writes:[4]

> [...] That our reasoning carries us to New Z is The Repugnant Conclusion.

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[2] D. Parfit: *Reasons and Persons*, Oxford: Clarendon Press, 1984, pp. 338, 419-441.

[3] A "reasonable" principle of distribution, in this context, is a principle which implies that if one of two outcomes with the same people has a greater total of welfare and it has welfare more equally distributed, then it is better.

[4] J. Griffin: *Well-Being. Its meaning, Measurement and Moral Importance*, Oxford: Clarendon Press, 1986, p. 340 (note 27).

> But does it? [...] there is another possibility confined entirely to the reasoning about beneficence. Parfit's argument seems implicitly to employ a totting-up conception of measuring well-being; it treats well-being as measurable on a single continuous additive scale, where low numbers, if added to themselves often enough, must become larger than any initial, larger number. But this seems not true in prudential cases, and it would seem likely that this incommensurability in prudential values would get transferred to interpersonal calculation. Perhaps it is better to have a certain number of people at a certain high level than a very much larger number at a level where life is just worth living. Then we might wish to stop the slide [...] at that point along the line where people's capacity to appreciate beauty, to form deep loving relationships, to accomplish something with their lives beyond just staying alive ... all disappear.

Griffin points to an implicit assumption as to the measurement of welfare. He claims that Parfit's arguments to the effect that we cannot avoid the Repugnant Conclusion implicitly assume welfare to be measurable on a continuous additive scale satisfying what is known as the *Archimedean* property of real numbers: for any positive number x, no matter how small, and for any number y, no matter how large, there exists an integer n, such that $nx \geq y$. This simply means that any two (positive) levels of welfare are commensurable, i.e. their ratio is not infinite.

But reflection on the measurement of welfare suggests that this assumption is not fulfilled. Presumably, then, a certain low level ***z*** could be infinitely small compared with other, higher levels, for instance ***a***. And it would seem to follow that this level could never add up to the high level, that is, the total *n**a*** would necessarily be greater than the total *m**z***, no matter how large *m* is. Therefore, the Utilitarian Total Principle does not imply the Repugnant Conclusion. At least this is how I shall understand Griffin's suggestion.

Somewhat strangely, no one appears to have taken this suggestion seriously. Roger Crisp is an exception.[5] He explicitly draws out the consequence that some form of discontinuity will block that the Repugnant Conclusion follows from Total Utilitarianism. However, he also identifies this position as a version of Parfit's Lexical View.[6] This seems to me a confusion of two clearly distinct positions. As I understand the Lexical View, there is a standard (Archimedean) scale of welfare; but on this scale we then determine two levels, such that lives above the higher level are assigned a *weight* which lexically dominates the weight of lives below the lower level.

The trouble with this view is that it requires some justification to claim that some persons' welfare should weigh differently than others'. It can hardly be said to be a concern for beneficence to assign *less* weight to low levels. And the weights have nothing to do with considerations of equality: In the outcomes in question, there is complete equality. It is precisely because it avoids this problem that Griffin's suggestion deserves attention.

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[5] R. Crisp: *Ideal Utilitarianism: Theory and Practice*. DPhil. Thesis, Oxford University, 1988, pp. 177-78.

[6] Parfit, *op.cit.*, p. 188.

Griffin introduces two superiority relations that could account for welfare not being measurable on a scale fulfilling the *Archimedean* property. One is *trumping*: "any amount of *A* outranks any amount of *B*" and the other is the weaker *discontinuity*: "enough of *A* outranks any amount of *B*".[7] He considers the latter more plausible and therefore his argument is based on discontinuity.

In this paper, I shall present some general results on the properties of this value superiority relation between objects. As we shall see, it behaves very differently, depending on whether value is additive or not. In an additive context, discontinuity collapses into trumping. And in a non-additive context, discontinuity – perhaps counter intuitively – does not imply a radical value difference at all. I then spell out the consequences of these results for different interpretations of Griffin's suggestion regarding population ethics.

2. Superiority in Value when Value is Additive

This section and the next build on a seminal paper by Arrhenius & Rabinowicz.[8] I shall merely draw out some consequences, which are more or less implicit in their work. The framework is this: Suppose there is a

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[7] Griffin, *op.cit.*, pp. 83-86.

[8] G. Arrhenius & W. Rabinowicz: "Millian Superiorities", *Utilitas* 17 (2005): 127-146.

countable set of objects. I assume that there is a *concatenation procedure* by which it is possible to form a new object by conjoining a finite number of separate objects into one whole. This includes the possibility of conjoining an object ***e*** a finite number of times *m* with an object exactly like itself; such an object is designated by '*m**e**'*. The domain is closed under concatenation.[9]

On the domain of objects, there is a *weak betterness relation*, – is at least as good as –. *Strict betterness* and *equivalence* relations are defined in the usual way.[10] This weak betterness relation is assumed to be *transitive*[11] and *complete*[12]. It is further assumed that concatenation is value-increasing, i.e. for all objects ***e*** and ***e'***, the whole consisting of ***e*** and ***e'*** is better than ***e***. In particular, 'self-concatenation' is value-increasing, i.e. for all objects ***e*** and all *m*>1, *m**e*** is better than (*m*-1)***e***.

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[9] It is assumed that concatenation is associative, which means that we get the same whole from concatenating any three objects, regardless of the order in which they are concatenated

[10] That is: ***e*** is better than ***e'***, if and only if ***e*** is at least as good as ***e'***, and ***e'*** is not as least as good as ***e***; and ***e*** is equivalent to ***e'*** if and only if ***e*** is at least as good as ***e'***, and ***e'*** is at least as good as ***e***.

[11] That is: for all objects ***e***, ***e'***, ***e''***: if ***e*** is at least as good as ***e'***, and ***e'*** is at least as good as ***e''***, then ***e*** is at least as good as ***e''***.

[12] That is: for all objects ***e***, ***e'***; either ***e*** is at least as good as ***e'*** or ***e'*** is at least as good as ***e***.

Now, we can define the relevant superiority relations corresponding to Griffin's *trumping* and *discontinuity* (I shall use Arrhenius' and Rabinowicz' terminology from now on):

DEFINITION 1: An object ***e*** is *superior* to an object ***e'*** if and only if, for all positive integers *n*, ***e*** is better than *n****e'***.

DEFINITION 2: An object ***e*** is *weakly superior* to an object ***e'*** if and only if, for some positive integer *m* and all positive integers *n*, *m****e*** is better than *n****e'***.

First, I shall assume that value is *additive with respect to concatenation*, i.e. the value of a concatenated whole is the sum of the value of each of its constituents. For this, the following condition is the principal necessary condition:[13]

INDEPENDENCE: An object ***e*** is at least as good as ***e'***, if and only if ***e*** concatenated with any object is at least as good as ***e'*** concatenated with that object.[14]

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[13] Cf. D. H. Krantz., D. R. Luce, P. Suppes and A. Tversky: *Foundations of Measurement*. Vol. 1: *Additive and Polynomial Representations*. San Diego: Academic Press, 1971, pp. 73-74.

[14] Arrhenius & Rabinowicz only assume the 'only if'-part in their *Independence*-condition – that is all they need for their *Observation 2*.

Hence, replacing ***e'*** by ***e*** in any whole results in a whole that is at least as good.

Consider the *Archimedean Condition* that for all ***e***, ***e'*** there exists a positive integer *n* such that ***ne'*** is at least as good as ***e***. This condition is called *Archimedean* because it corresponds to the *Archimedean* property of real numbers. Since the *Archimedean* property is true of the real numbers, the *Archimedean Condition* is necessary for measurement in real numbers. If the Archimedean condition holds and the domain is sufficiently rich to ensure the solvability condition that if ***e*** is better than ***e'***, then there exists some ***e''*** such that ***e*** is equivalent to the whole consisting of ***e'*** and ***e''***, then the betterness relation could be represented by a real-valued function which is additive with respect to concatenation.[15]

Suppose that ***e*** is superior to ***e'*** and *Independence* holds. Superiority violates the *Archimedean Condition* – superiority is precisely defined as the condition that there is no number such that ***ne'*** is at least as good as ***e***. Consequently, the value ratio between ***e*** and ***e'*** is *infinite* and cannot be measured by any real number.[16]

Consider next weak superiority. In his discussion of measurement of well-being, Griffin (1986, p. 85) says about weak superiority (which he calls 'discontinuity') that it brings with it

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[15] Krantz *et al.*, *ibid*.

[16] Cf. Krantz *et al*., *op.cit*., pp. 271-272.

> the suspension of addition; [...] we have a positive value that, no matter how often a certain amount is added to itself, cannot become greater than another positive value, and cannot, not because with piling up we get diminishing value or even disvalue (though there are such cases), but because [it is] the sort of value, that, even when remaining constant, cannot add up to some other value.

It is not part of the definition of weak superiority that value is additive. However, Griffin seems here to assume that it is. Weak superiority likewise violates the *Archimedean Condition*. However, weak superiority is further assumed to imply that the inferior value *can* add up to *some* amount of the weakly superior value. It is only when the amount of the weakly superior value is sufficiently large ("enough") that the inferior value can never add up to this amount – addition is then "suspended".

I shall demonstrate that this picture cannot be upheld. If ***e'*** cannot add up to *m**e***, it cannot even add up to ***e***. In other words, if we assume *Independence*, then weak superiority collapses into superiority:

OBSERVATION 1: *Independence* implies that if some element ***e*** is weakly superior to another ***e'***, then ***e*** is also superior to ***e'***.

PROOF: Assume, for *reductio*, that some element ***e*** is weakly superior to another element ***e'***, but not superior to it. The fact that ***e*** is weakly superior to ***e'*** means that there is some *m*, such that *m**e*** is better than any number of ***e'***-elements.The fact that ***e*** is not superior to ***e'*** means that there is some *q*, such that ***e*** is not better than *q**e'***. Assume, for all *n*=2, 3, …, that (*n*-1)***e*** is not better than $(n\text{-}1)q\boldsymbol{e'}$. *Independence* then implies that $n\boldsymbol{e}$ is not better than $(n\text{-}1)q\boldsymbol{e'}$ concatenated with $\boldsymbol{e}$. Since $\boldsymbol{e}$ is not better than $q\boldsymbol{e}$, *Independence* implies that $(n\text{-}1)q\boldsymbol{e'}$ concatenated with $\boldsymbol{e}$ is not better than $nq\boldsymbol{e'}$. By mathematical induction, it then follows that $n\boldsymbol{e}$ is not better than $nq\boldsymbol{e'}$. But then it cannot be the case that that there is some m, such that $m\boldsymbol{e}$ is better than any number of $\boldsymbol{e'}$-elements.

Arrhenius and Rabinowicz prove that, if in a decreasing sequence $\boldsymbol{e}_1, \ldots, \boldsymbol{e}_n$ the first element is superior to the last one, then *Independence* implies that some element in the sequence is superior to its immediate successor.[17] So in this case we cannot come from the superior object to the inferior object through a number of steps where each object in the sequence is only marginally worse than its immediate predecessor. At least one step is itself a step to something drastically worse – as a matter of fact, we know from above that it is a step to something infinitely worse, since the difference cannot be measured by any real number.

3. Superiority in Value when Value is Non-Additive

However, suppose we give up additivity, i.e. give up *Independence*. For this case, Arrhenius and Rabinowicz prove: for any two objects $\boldsymbol{e}$ and $\boldsymbol{e'}$, where $\boldsymbol{e}$ is weakly superior to $\boldsymbol{e'}$ without being superior to it, the domain must contain a finite decreasing sequence of objects in which the first

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[17] Arrhenius & Rabinowicz, *op.cit.*, *Observation 2*.

element is superior to the last one, but no element is superior to its immediate successor.[18]

We know from OBSERVATION 1 that the requirement that weak superiority does not collapse into superiority is inconsistent with *Independence*. And as Arrhenius and Rabinowicz note, if *Independence* is denied, then it becomes possible that by concatenating some object ***e'*** to itself any number of times, the marginal value of each contribution, though always positive, converges to zero, such that there is a *finite* upper limit to the aggregated value. I should like to demonstrate that this will *necessarily* be the case.

Remember that we are dealing with a transitive and complete weak betterness relation defined on a countable set of objects on which a concatenation operation is defined, such that the domain is closed under concatenation. Since the domain is a countable set, the betterness relation can be represented ordinally by a real-valued function *V*, such that

e is as least as good as ***e'*** if and only if $V(\boldsymbol{e}) \geq V(\boldsymbol{e'})$.[19]

Now the following can be proved:

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[18] Arrhenius & Rabinowicz, *op.cit.*, *Observation 1*.

[19] Cf. Krantz *et al.*, *op.cit.*, p. 39 (Theorem 2.1).

OBSERVATION 2: Suppose it is the case that some object ***e*** is weakly superior to another ***e'*** without being superior to it. Then the sequence *V*(***e'***), *V*(2***e'***), *V*(3***e'***), … has an upper bound.

PROOF: Since ***e*** is weakly superior to ***e'***, there is some *m*, such that *V*(*m**e***) is greater than *V*(*n**e'***), no matter how big *n* is. It follows immediately that the sequence *V*(***e'***), *V*(2***e'***), *V*(3***e'***), … has *V*(*m**e***) as upper bound.

Hence, weak superiority is not a sign of a large difference between the superior and the inferior object, but rather dependent on how the value of self-concatenation of the inferior object develops. We also know[20] that if some object ***e*** in a decreasing sequence is weakly superior to another ***e'*** without being superior to it, then the domain also contains some object that is superior to ***e'***. In fact, any object with a value above the upper bound of the *V*(***e'***), *V*(2***e'***), *V*(3***e'***), … will be superior to ***e'***. Thus, under these circumstances, not even superiority is a sign of a radical difference.

It could even be the case that some object ***e*** is superior to ***e'***, but not to some object ***e''***, which is worse than ***e'***, because the aggregated value of self-concatenation of ***e''*** has a higher upper bound than that of ***e'*** or even no upper bound and hence an aggregated value that could exceed the value of ***e***. Note also that if self-concatenation of ***e''*** has a higher upper bound than that of ***e'***, then we have a case where ***e''*** is weakly superior to ***e'*** even though ***e''*** is worse than ***e'***.

[20] From Arrhenius & Rabinowicz, *op.cit.*, *Observation 1*.

Arrhenius and Rabinowicz also prove that if in a finite sequence of objects the first element is weakly superior to the last element, then there exists at least one element that is weakly superior to its immediate successor.[21] Given my observations above, there is a simple and informative proof of this observation:

PROOF: Suppose *Independence* is fulfilled. Because of my OBSERVATION 1, weak superiority collapses into superiority and we know[22] that in a decreasing sequence where the first element is superior (and therefore also weakly superior) to the last element, then some element in sequence is superior (and therefore also weakly superior) to it immediate successor. And if *Independence* is not fulfilled, we know from my OBSERVATION 2 that the last element, concatenated by an element like itself any number of times, has an upper bound. The preceding element, concatenated any number of times by an element like itself, either has an upper bound which is higher than this or has no upper bound, in which case it is weakly superior to the last element (because there will be some number of this element which is better than any number of the last) and the proof is done, or it has an upper bound that is lower or equal to the bound of the last element, in which case it is not weakly superior to it (indeed, if the upper bound is lower the last element would be weakly superior to the preceding one). In the latter case, we can repeat the procedure until we are either done

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[21] Arrhenius & Rabinowicz, *op.cit.*, *Observation 3*.

[22] From Arrhenius & Rabinowicz, *op.cit.*, *Observation 2*

or finally reach the next-to-the-first element, which in that case must have en upper bound that is lower or equal to the bound of the last element; but since the first element by hypothesis is weakly superior to the last one, it is also weakly superior to this one.

Arrhenius and Rabinowicz are somewhat surprised by their result, because they start out with the intuition that both superiority and weak superiority are drastic differences in value. Whereas Rabinowicz is willing to accept that an element can be weakly superior to another even though it is only marginally better, Arrhenius[23] sticks to the intuition and takes the line that Rabinowicz' and his results provide an argument against superiority and weak superiority in all contexts where it is possible to construct a sequence of objects in which the value differences between adjacent objects are marginal.

But as my results show, weak superiority does not depend on the difference between elements, but solely on how the aggregated value of self-concatenation develops. Hence, even in a decreasing finite sequence in which each consecutive element is only *marginally* worse than the immediately preceding one, weak superiority can obtain; and even an element which is worse than another might be weakly superior to it. For the same reason, weak superiority between the *extrema* of a finite sequence does not mean that the last element is radically worse than the point of

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[23] See here also G. Arrhenius: "Superiority in Value", *Philosophical Studies* 123 (2005): 97-114.

departure. Moreover, if we have a finite decreasing sequence of objects in which the first element is superior to the last one, and some element in the sequence is weakly superior to another one, then not even superiority is a radical difference in value.

Arrhenius further claims that superiority and weak superiority share some problems when we consider outcomes that involve both superior and inferior objects.[24] Consider three objects ***e***, ***e'*** and ***e''***, where ***e*** is only marginally better than ***e'***, and ***e'*** is clearly better than ***e''***. Assume further that ***e*** is superior or weakly superior to ***e'*** (and in the latter case, let *n* be a number such that *n**e*** is better than any number ***e'***). Compare a whole, ***a***, consisting of *n**e***, with another whole, ***b***, consisting of *n**e*** and *m**e''***, where *m* is much greater than *n*. By the assumption of value increasingness, ***b*** is better than ***a***. Consider now a third whole, ***c***, consisting of (*n*+*m*)***e'***. Since ***e*** (or *n**e***) is superior to ***e', a*** is better than ***c***. However, Arrhenius says, compare ***b*** and ***c***: since the loss of getting ***e'*** instead of ***e*** is only marginal, and the gain from getting ***e'*** instead of ***e''*** is bigger, there should be some sufficiently large *m*, such that ***c*** is better than ***b***; and then, by transitivity, we would have that ***c*** is better than ***a***. Hence, the notions of superiority and weak superiority seem to imply a contradiction in this case.

But, taken on its face value, Arrhenius' reasoning is mistaken. If ***e*** is superior (or weakly superior) to ***e'***, then the loss of ***e*** (or the loss of having less than *n**e***) cannot be compensated by any number of ***e'***. Therefore, ***b*** is

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[24] Arrhenius, *op.cit.*, pp. 108-109.

better than ***c***, and there is no contradiction. When he says that there is marginal loss for *each* ***e***-object that has been exchanged for an ***e'***-object, but a bigger gain for *each* ***e''***-object that has been exchanged for an ***e'***-object, he appears to assume *Independence*. But if *Independence* is fulfilled, and ***e*** is superior to ***e'***, the value difference between ***e*** and ***e'*** cannot be marginal, which it is by hypothesis in the example.

However, Arrhenius might still have a point. When he says that it is hard to deny that there is some *m* such that the smaller number of smaller losses is compensated for by the greater number of greater gains, he could be understood as implying that, given the marginal difference between ***e*** and ***e'***, it is implausible that the loss of *n**e*** (or even ***e***) cannot be compensated by a sufficiently large number *m* ***e'***-objects; otherwise, the marginal value of adding extra ***e'***-objects would – implausibly – diminish extremely rapidly. This may be true, in which case ***c*** would be better than ***b*** (which is better than ***a***); but if it is, ***e*** cannot be superior or weakly superior to ***e'***, and so ***a*** cannot at the same time be better than ***c***. Thus, even though there is no contradiction, this reasoning could support his initial claim that superiority relations are implausible in these contexts.

4. Weak Superiority and the Repugnant Conclusion: The Case of Infinite Standard Sequences on the Inferior Values

Griffin's main idea is that no amount of certain less important values can ever compensate a substantial loss of certain more important and genuine values. The underlying picture here is that welfare depends on the degree to which a number of prudential values are realised.[25] More precisely, I shall assume that welfare is measured by the sum of the value contribution from each value. And if one of the important values is realised to a sufficient degree, its value contribution is such that the contribution from an unimportant value never can add up to it, no matter how much it is realised.

Griffin gives an example, where an unimportant value is a sort of residual value to an important one, such that when the latter is lost, we might get the former:[26] The important value is "appreciation of beauty". If we gradually reduce the degree to which this value is realised, we shall eventually reach a point, where it is lost. We might instead have "kicks of kitsch" but they are different, that is, they represent a different value, which still gives a positive contribution but one that is inferior to the contribution from genuine appreciation of beauty.

Griffin's suggestion is based on the idea that weak superiority between valuable objects is a more plausible condition than superiority. But since his suggestion also implies a context where value is additive, we should expect from OBSERVATION 1 that weak superiority collapses into superiority. However, to be able to apply OBSERVATION 1 on this context, I need to set up a slightly more complicated apparatus.

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[25] Cf. the list in Griffin, *op cit.*, p. 67.

[26] Griffin, *op.cit.*, pp.86-87.

I assume a list of prudential values *A*, *B*, *C*, … . In a given life, each of these values is realised to a certain degree which I shall assume can be measured by a non-negative number. Thus, there is a domain of possible lives, $\boldsymbol{L}=\{\boldsymbol{l}_1, \boldsymbol{l}_2, \boldsymbol{l}_3, \ldots\}$, where each life is represented by a vector (*a*, *b*, *c*, …) and *a*, *b*, *c*, … are non-negative numbers describing the degree to which each of the values *A*, *B*, *C*, … is realised in this life.

Like before, there is a *weak betterness relation*, – is at least as good as –, defined on this domain, which is assumed to be *transitive* and *complete*. This relation embodies global preferences over the domain of possible lives, which Griffin considers basic for the measurement of welfare.

Some values might really be disvalues, giving a negative contribution to overall welfare. In this context, I shall only consider values giving positive or zero contribution to overall welfare. However, I shall allow for the fact that the marginal contribution from each positive value diminishes the more it is realised.

Next, I shall assume some necessary conditions for value contributions to be additive, such that the welfare of a life, $w(\boldsymbol{l})$ where $\boldsymbol{l}=(a, b, c, \ldots)$ could be measured by the sum of contributions $w_A(a)+w_B(b)+w_C(c)+\ldots$ from each of the values. Here, I draw on what is known as *additive conjoint measurement*.[27] Additive conjoint measurement does not rely on a simple concatenation procedure like the one outlined above, with respect to which

[27] Cf. Krantz *et al., op.cit.*, Chapter 6.

it is additive. In order to establish additivity, it simulates concatenation in a more complicated way.[28] The first necessary condition is a form of independence known as *strong separability*:

DEFINITION 3: Consider some subset of values, say *P* and *Q*, and let the degree of realisation of each of the remaining values be kept constant ($\underline{a}$, $\underline{b}$, …, $\underline{o}$, $\underline{r}$, $\underline{s}$, …). The betterness relation will rank alternative combinations (*p*, *q*) of the degree to which *P* and *Q* are realised, given this fixed choice of the degree of realisation of the remaining values. If this ranking is the same for all possible fixed choices, the subset of values *P* and *Q* are said to be *separable* in the betterness relation. If any arbitrary subset of values is separable in this way, the betterness relation is said to be *strongly separable*.

If the betterness relation is strongly separable, it induces a transitive and complete betterness relation on each subset of values. In other words, we can then evaluate the betterness of each subset of values independently from the other values.

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[28] In the standard framework, it is assumed that the set of possible lives, ***L***, is a product set. This means that the values in a life are independently realisable, i.e. that that the domain contains every possible combination of degrees of realisation of values. However, this is not a condition which is necessary for the additive representation as such.

The other necessary condition I shall introduce is the existence of standard sequences for each value. A *standard sequence* defines value differences having non-zero, equal spacing in the intended numerical representation of the value contribution from each value. Consider again the values *P* and *Q*. Arbitrarily, define some unit q_1 on *Q*. Now, find some degree p_1, such that $(p_1, 0)=(0, q_1)$.[29] (Note that this only works if *P* and *Q* are both important or both unimportant. Hence, I shall assume that there are at least two values on each level). Then, find some degree p_2, such that $(p_2, 0)$ is equivalent with (p_1, q_1). Go on and find some degree p_3, such that $(p_3, 0)$ is equivalent with (p_2, q_1). Similarly, define p_4, p_5, … . Now, we have defined a standard sequence on value *P*:

DEFINITION 4: A sequence of degrees p_i, p_{i+1}, …, i=1, 2, … , of some value *P* is a *standard sequence* if and only if there exists q_1, q_2 on some other value *Q* such that q_1 is better than q_2 and for all i=1, 2, …, (p_i, q_1) is equally as good as (p_{i+1}, q_2). A standard sequence can be either finite or infinite.

A similar procedure can be used on *Q*. And we can go on adding definitions of fractions of the chosen unit.

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[29] To be sure this can be done, we need to assume a solvability condition, cf. Krantz *et al., op.cit.*, p. 301. This is another structural condition, which is not necessary for the additive representation.

The plausibility of these conditions depends of course a lot on whether it is possible to describe and individuate values in a way such that the contribution of any subset of values, as derived from our basic preferences over possible lives, is independent on the degree to which other values are realised. I shall not here attempt to argue that these conditions are in fact fulfilled. The point is merely to point out that these conditions are necessary for the argument Griffin wants to make, and to spell out the consequences.

The standard sequence p_1, p_2, p_3, … can be understood as additive self-concatenation of p_1, where p_2 is a whole consisting of p_1 concatenated with an object like itself, and where also q_1 is defined as an object like p_1. And given strong separability and a standard sequence p_1, p_2, p_3, … defined on P using q_1, q_2, q_3, … on Q, it follows straightforwardly from the definition of standard sequences that *Independence* will be fulfilled for this standard sequence on P, such that, for all non-negative integers n, n' and m where $n \geq n'$, p_n is at least as good as $p_{n'}$ if and only if p_{n+m} is at least as good as $p_{n'+m}$.[30]

Suppose now, as Griffin suggests, that A is weakly superior to B in the sense that, for any standard sequences a_1, a_2, a_3, … and b_1, b_2, b_3, …,[31] there is some m such that (a_m, 0) is better than (0, b_n), no matter how big n

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[30] In other words, *if* a numerical representation was possible (which I have not yet assumed), we would have $w_P(p_2)=2w_P(p_1)$, $w_P(p_3)=3\ w_P(p_1)$, …

[31] Note that these standard sequences are not defined relative to each other.

is. In the context of additive conjoint measurement, the *Archimedean* condition can be stated thus: *every strictly bounded standard sequence is finite*.[32] But as weak superiority is defined here, it violates the *Archimedean* condition, because the standard sequence b_1, b_2, b_3, … is infinite and still strictly bounded by (a_m, 0). Hence, the difference between a_m and b_1 is infinitely large and cannot be measured by any real number. This is precisely what Griffin needs.

However, we are now in a position where we can apply OBSERVATION 1 and demonstrate that weak superiority collapses into superiority: if (a_m, 0) is better than (0, b_n), no matter how big n is, then it follows from OBSERVATION 1 that (a_1, 0) is better than (0, b_n), no matter how big n is. In other words, even the smallest degree to which *A* can be realised will be better than *B*, no matter the degree to which it is realised. Consequently, contrary to his inclination, if Griffin wants to keep the idea that some levels of welfare do not add up to others, he is bound to base his argument on superiority rather than on weak superiority.

Even so, we might still have a credible view which might avoid the Repugnant Conclusion. I shall make some of its implications clear. Consider a sequence of lives with decreasing of levels of welfare ***a***, ***b***, ***c*** …, ***z***. Given that we accept the Utilitarian Total Principle, a concatenation procedure is defined (i.e. summing up welfare levels), the betterness relation is given and the *Independence* condition is satisfied. Griffin

[32] Cf. Krantz *et al., op.cit.*, p. 253.

suggests that some number of people *n* at some level, say ***m***, is weakly superior to the level **z**. But then it follows from OBSERVATION 1, firstly, that ***m*** is superior to ***z***; in other words, it would be the case that even one person at ***m*** outranks any number of lives at **z**.

Furthermore, we know[33] that if we have a decreasing sequence ***m***, ***n***, …, ***z*** where ***m*** is superior to ***z***, then there will be some level in the sequence which is superior to its immediate successor. In other words, one person living at the lowest level before the discontinuity sets in is superior to any number of people living at ***z***. Call this level ***y***. Suppose there is some *n* such that *n****y*** is equally good as ***m***. Then the same consequence would follow from my OBSERVATION 1: Since ***y*** is weakly superior to ***z*** (because ***m*** and therefore also *n****y*** is superior to ***z***), it follows that ***y*** is superior to ***z***.

Strictly speaking, this view does not avoid the Repugnant Conclusion as I have stated it. I shall assume that welfare is measurable on a scale that has an extension of the real numbers with infinitesimal numbers.[34] I shall not go into technical details about infinitesimal numbers.[35] It suffices with the

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[33] From Arrhenius and Rabinowicz' *Observation 2*

[34] Alternatively, we could imagine that welfare is measured on two dimensions, cf. M. Hausner: Multidimensional Utilities. In R. M. Thrall, C. H. Coombs and R. L. Davis (eds.): *Decision Processes*, John Wiley, 1954, pp. 167-80.

[35] There is a rigorous treatment in A. Robinson: *Non-Standard Analysis*, Revised edition, Amsterdam: North-Holland, 1974. As for measurement, see L. Narens: "Measurement without Archimedean Axioms", *Philosophy of Science* 41 (1974):

intuitive understanding that adding an infinitesimal number to another infinitesimal number results in an infinitesimal number, and consequently, that multiplication of an infinitesimal number with an integer results in another infinitesimal number. On this scale, it is still the case that, for any positive finite level of welfare, $\boldsymbol{y}$, however close to zero, a population of m people at $\boldsymbol{y}$ is better than n people at $\boldsymbol{a}$, provided $m\boldsymbol{y}>n\boldsymbol{a}$. It is just that below such level $\boldsymbol{y}$, however low, there are lives definitely worse than $\boldsymbol{y}$ but still worth living – these are the ones measured by infinitesimal small levels. So maybe the level $\boldsymbol{y}$ is not so bad after all.[36]

Whether this is a credible view depends on how the zero on the scale is determined,[37] and where the discontinuity sets in. Like Parfit, Griffin appears to have a zero for the scale of welfare in mind, which is something like the level where there is no point of living the life, a life with neither positive nor negative value at all; [38] and implicitly, this zero is also the level

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374-393, L. Narens: "Minimal Conditions for Additive Conjoint Measurement and Qualitative Probability", *Journal of Mathematical Psychology* 11 (1974): 404-430 and H. J. Skala: *Non-Archimedean Utility Theory*. Dordrecht: D. Reidel, 1975.

[36] I owe this interpretation to a communication with John Broome.

[37] Cf. John Broome: Weighing Lives, Oxford University Press, 2004, p. 138. This point is largely overlooked.

[38] Griffin, op.cit., pp. 130-131, 345 (note 12).

where the existence of the person is indifferent from the point of view of the Total Principle.[39]

As regards the discontinuity, we saw above that even the smallest degree to which some superior value *A* can be realised will be better than some inferior value *B*, no matter the degree to which it is realised. This means that a life where just one superior value is realised to the smallest possible degree is infinitely better that a life where no superior values are realised. Interestingly, Griffin seems to accept this consequence, when he says: [40]

> Then we might wish to stop the slide [...] at that point along the line where people's capacity to appreciate beauty, to form deep loving relationships, to accomplish something with their lives beyond just staying alive ... *all* disappear.

Given this more explicit statement, we can now evaluate the repugnant-like conclusion: For any positive finite (i.e. non-infinitesimal) level of welfare, ***y***, however low, a population of *m* people at ***y*** is better than *n* people at ***a***, provided *m**y***>*n**a***. It means that a sufficiently large number of persons with lives just barely realising one important value in life will represent a greater total and therefore be better than some number of persons with a very high welfare. This still seems to me a rather repugnant conclusion.

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[39] Conceptually, however, these are two different questions. Cf. Broome, *op.cit*, Chapter 14.

[40] Griffin, *op.cit.*, p. 340 (note 27), my italics.

Another troublesome implication stems from the fact that even one person living at a finite level as close to zero as we want is better than any number of people living at a positive infinitesimal level. It means that the view implies a version of what Parfit calls the Absurd Conclusion: It is better that there live no people at all than a number of people with infinitesimal low welfare and one person in suffering at any finite negative level. Like Parfit, many would find this implication absurd.

5. Weak Superiority and the Repugnant Conclusion: The Case of Finite Standard Sequences on the Inferior Values

Suppose next that we give up the condition necessary for violating the Archimedean Condition, i.e. the existence of infinite standard sequences on the inferior values. There is a transitive and complete weak betterness relation defined on the domain of possible lives, $\boldsymbol{L}=\{\boldsymbol{l}_1, \boldsymbol{l}_2, \boldsymbol{l}_3, \ldots\}$, where each life is represented by a vector (a, b, c, …) and a, b, c, … are non-negative numbers describing the degree to which each of the values A, B, C, … are realised in this life. Assume that weak betterness at least fulfils *strong monotonicity* such that, for all pairs of lives, if one life has all values realised to at least the same degree as another, and at least one value is realised to higher degree, then it is better. It might or might not fulfil strong separability. In the former case, it might even allow for definition of standard sequences; however, any strictly bounded standard sequence is finite.

If in this framework *A* is weakly superior to *B*, it means that there is some $\underline{a}$ such that if *A* drops below $\underline{a}$, no degree of *B* can ever compensate this loss in value (other values kept constant). However, this is compatible with the superior and the inferior life being measurable on the same real-valued scale, becasue the contribution from the inferior value has a finite upper bound. If we were to apply the Utilitarian Total Principle,[41] it is clear that *n* people on the superior level always can be outweighed by a sufficiently large number *m* on the inferior level. Hence, in this case, the Repugnant Conclusion could not be avoided.

However, even though Griffin's suggestion appears to be based on the idea that values are additive, he is in fact rather sceptical about measuring welfare on a cardinal scale.[42] Thus, it is possible to understand how weak superiority between in prudential values would get transferred to interpersonal calculation in another way. Just as the comparison of lives is a matter of basic preferences, not of calculations based on other sources, comparisons of populations will have to be a matter of basic rankings.[43] If

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[41] However, in this case, the framework described so far does not provide a cardinal scale that would allow summing up welfare.

[42] Cf. Griffin, *op.cit.,* pp. 88, 98-102.

[43] This is how G. Arrhenius: *Future Generations. A Challenge for Moral Theory*, Uppsala University, 2000 understands Griffin (pp. 97-97). The interpretation is also apparent in R. Crisp: "Utilitarianism and the Life of Virtue", *The Philosophical Quarterly* 42 (1992): 139-160.

this is the case, there is no route to determining the most beneficent outcome through simple summation. Then Griffin's suggestion

> Perhaps it is better to have a certain number of people at a certain high level than a very much larger number at a level where life is just worth living.

could be interpreted as such a basic ranking where the weak superiority does not collapse.

Consider a sequence of lives with decreasing of levels of welfare ***a***, ***b***, ***c*** …, **z**. I assume that there is a concatenation procedure, such that we can concatenate lives into wholes (that is, populations) and that the domain is closed under concatenation. I also assume that a weak betterness relation, which is transitive and complete, is defined on the domain. In this framework, the ranking can be stated as: There is some level ***p***, which is weakly superior to **z**, without being superior to it. This means that there is some number *m* such that, for all positive integers *n*, *m**p*** is better than *n***z**.

We know from section 1 that the betterness relation on this domain can be represented ordinally by a real-valued function, *V*. And we know from OBSERVATION 2 that if ***p*** is weakly superior to **z** without being superior to it, then the increasing sequence *V*(**z**), *V*(2**z**), *V*(3**z**), … has an upper bound.

Thus, the basic ranking in this context implies that adding more people at **z** has diminishing marginal value that converges to zero. It makes the value of a person at **z** depend on how many other people there are at this level.

And this is implausible. I can think of no reason having to do with beneficence why one out of two persons at the same level of welfare should have more weight than the other. This is my main objection to this view.

Arrhenius states another objection, namely that this view either implies the *Mere Addition Paradox*[44] or else it violates what he calls *The Inequality Aversion Condition*: for any triplet of welfare levels ***a***, ***y***, ***z*** and for any population *n**a**,* there is some number *m*, such that the perfectly equal population (*m*+*n*)***y*** is at least as good as the population combined of *n**a*** and *m**z***. The combined population *n**a*** and *m**z*** is better than the population *n**a***. If we assume that the view complies with *The Inequality Aversion Condition*, the equal population (*n*+*m*)***y*** will be at least as good as the population combined of *n**a*** and *m**z***. By transitivity, the equal population (*m*+*n*)***y*** is then at least as good as *n**a***. But the view is supposed to imply that *n**a*** is better than (*m*+*n*)***y*** in the cases where ***a*** is weakly superior to ***y***. Hence, the view cannot comply with *The Inequality Aversion Condition*.

The Inequality Aversion Condition is widely assumed to be very plausible.[45] However, if one share the basic ranking that a certain number of people, *n*, at a certain high level, ***a***, is better than a very much larger number (*n*+*m*) at a level ***y*** where life is just worth living, it might not be unreasonable to deny *The Inequality Aversion Condition*. It should be accepted that a population combined of *n**a*** and *m**z***, where ***z*** is slightly

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[44] Cf. Parfit, *op.cit*., Chapter 19.

[45] In fact, it is a weak statement of the requirement I mentioned in note 3.

worse than ***y***, is better than ***na*** alone. But it might be claimed that, in spite of the equality obtained by the move from ***na*** combined with *m**z*** to (*n*+*m*)***y***, no number of people gaining slightly more than ***z*** can compensate the great loss of the *n* people. Therefore, I am not inclined to attach great weight to Arrhenius' objection.

6. A Different Suggestion

Let me briefly mention a suggestion, which I believe Jonathan Glover was the first to make:[46]

> But the concession that, other things being equal, there is value in extra happy people need not commit us to a simple policy of maximizing happiness. […] It is open to us to say that *one* thing we value is total happiness […] without simply adopting the total view. For we may decide that we value people's lives having various qualities (which would put them high on the scale of 'worth-while life') and that the absence of these qualities cannot be compensated for by any numbers of extra worth-while lives without them. […]
> So we can think that extra people with lives worth living are in themselves a good thing, without having to allow that there is always *some* number of people whose existence outweighs any particular impoverishment of life.

Parfit has made a similar suggestion.[47]

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[46] J. Glover, Jonathan: *Causing Death and Saving Lives*. Hammondsworth: Penguin, 1977, pp. 70-71

> Consider what I shall call *the best things in life*. These are the best kinds of creative activity and aesthetic experience, the best relationships between different people, and the other things which do most to make life worth living [...]
> Why is it so hard to believe that my imagined world Z […] would be better than a world of ten billion people, all of whom have an extremely high quality of life? This is hard to believe because in Z two things are true: people's lives are barely worth living, and most of the good things in life are lost. [...]
> What we might appeal to is […] *Perfectionism*. [...] We might claim that, even if some change brings a great net benefit to those who are affected, it is a change for the worse if it involves the loss of one of the best things in life.

The idea seems to be that, apart from the total of welfare, there is a separate consideration (Parfit calls it *Perfectionism*) concerning the various qualities in peoples' lives. And there is a superiority relation between them, such that a sufficiently high level of these qualities cannot be compensated by the existence of extra people without these qualities, whatever their number.

Whereas in the former section the evaluation was based on a basic ranking of outcomes, it is here based on two considerations that have to be weighed up against each other. Since the total of welfare approaches infinity when

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[47] Pp. 161-63 in D. Parfit: "Overpopulation and the Quality of Life", in: P. Singer (ed.): *Applied Ethics*, Oxford University Press, 1986, pp. 145-164.

the number of people increases, the level of sufficiently high quality will have to be infinitely better than the level just below of insufficiently high quality. Such a dramatic discontinuity might not be plausible.

Apart from that, this view will have the same implication as the Lexical View which I mentioned in the Introduction: it will put less weight to people at low level, and this might be hard to justify.

7. Conclusion

I have demonstrated that weak superiority behaves very differently, depending on whether value is additive or not. In an additive context, weak superiority collapses into superiority, which in this context is a radical difference. In a non-additive context, the inferior value has diminishing marginal value converging to zero, such that when the value increases, it approaches a finite upper bound. In the latter context, weak superiority is not a radical difference in value; and even superiority is not a radical difference in value if in-between the superior and the inferior object, some element is weakly superior to another one.

Often, this difference is overlooked. For instance, Griffin appears on the one hand to appeal to the larger plausibility of weak superiority between values and at the same to hint at infinite value differences. But it is not possible to have both. Also, I have demonstrated that in moving to an additive context, rather strong additional assumptions are needed.

Finally, I have also demonstrated the consequences of using weak superiority to block the Repugnant Conclusion in respectively an additive, a non-additive and a mixed context. In the first case (where weak superiority collapses), the Repugnant Conclusion is, strictly speaking, not avoided. It is just that below any finite level, there are lives definitely worse. In the second case, the implication is that adding more people at low levels has diminishing marginal value converging to zero. In the third case, there is a dramatic discontinuity in the consideration concerning the various qualities of peoples' lives.

None of these implications is plausible. However, since it is hard to come up with coherent theories in the field of population ethics, weak superiority perhaps retains some interest.